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An Energy-Based Method for Signal Compression and Reconstruction with Wavelets

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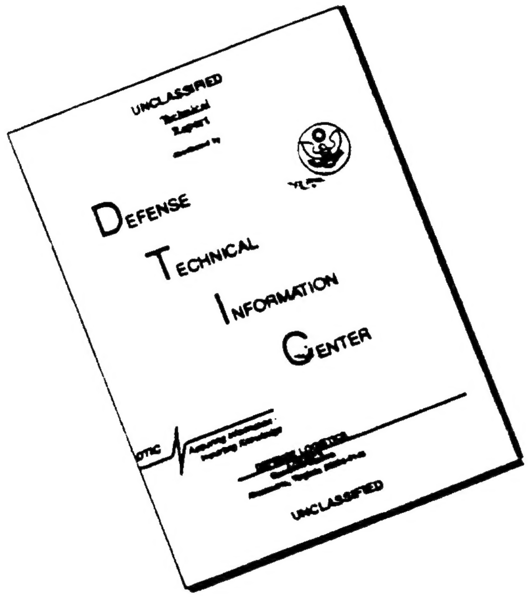

PREFACE

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Wavelets and wavelet transforms have recently emerged as a promising alternative to traditional Fourier-based spectral decompositions in a variety of signal processing applications. With the expected exponential increase in data traffic volume and the consequent overloading of storage capacity and transmission channels, the need for improved signal compression is essential.

A new energy-based method for selection of wavelet coefficients for signal compression is proposed. The number of coefficients selected from a particular level of the wavelet decomposition tree is proportional to the mean energy contained in the coefficients at that level. In experimental tests, this method provided significantly improved performance over conventional global thresholding-based wavelet selection techniques. The performance index used is the signal-to-error ratio, which is a measure of the quality of the reconstructed signal from its compressed representation compared to the original.

For highly nonstationary signals, a crossover effect is observed; that is, the energy-based selection is outperformed by the conventional method at higher percent retention levels. In such a situation, a segmentation strategy is proposed wherein the signal is decomposed into segments of similar characteristics prior to compression.

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LIST OF ABBREVIATIONS AND ACRONYMS

DCT	Discrete cosine transform
DFT	Discrete Fourier transform
FWT	Fast wavelet transform
K-L	Karhunen-Loeve transformation
NUWC	Naval Undersea Warfare Center
rms	Root mean square
SER	Signal-to-error ratio
SNR	Signal-to-noise ratio

AN ENERGY-BASED METHOD FOR SIGNAL COMPRESSION AND RECONSTRUCTION WITH WAVELETS

1. INTRODUCTION

Evolving remote sensing systems for today's undersea applications have resulted in ever-increasing demands for data storage and communication bandwidth. With an expected increase in the volume of data traffic and the consequent overloading of storage capacity and transmission channels, the need for signal compression is essential. Signal compression techniques can reduce the volume of data by exploiting these kinds of statistical redundancies in data: (1) spatial redundancy due to correlation between neighboring sensor measurements, (2) temporal redundancy due to correlation between successive segments of the same sensor signal, and (3) spectral redundancy between bands of multispectral signals.

In the past several decades, signal compression techniques have been mainly performed with traditional spectral decompositions such as the discrete Fourier transform (DFT) and discrete cosine transform (DCT). Signal compression by traditional techniques produces no discernible degradation in the reconstructed signal at low compression ratios (4:1 to 5:1). As demands for high compression ratios increase, a reconstructed signal of moderate quality can be obtained with traditional techniques at compression ratios up to 40:1 or so. However, for compression ratios higher than 40:1, the performance of traditional techniques deteriorate rapidly, and the reconstructed signal is degraded severely and is generally not useful (Hunt, 1978). To this end, researchers in the signal and image processing community have been searching for better techniques for data compression. In recent years, wavelets and wavelet transforms have emerged as a promising alternative to traditional spectral decompositions. Applications of wavelet-based methods for signal compression and reconstruction are discussed widely in the open literature (Strang, 1994). The most common technique used in wavelet decomposition is based on the global threshold method for selecting the coefficients of the wavelet transformed signal. When the global threshold method is used, the largest coefficients are retained as representative of the signal and all others are discarded.

This technical report proposes a novel method for selection of wavelet coefficients that uses a local threshold for the different resolution levels of the wavelet decomposition tree. This local threshold is based on the average energy of the wavelet coefficients in each decomposition level. The proposed method will ensure that the selected coefficients in the compressed signal are proportionally representative of the energy at each decomposition level. This report details the energy-based algorithm for selecting wavelet coefficients and presents experimental results of signal compression and reconstruction for marine biological sounds and underwater vehicle acoustic signals. Specifically, section 2 reviews signal compression processes using traditional and wavelet-based methods; section 3 introduces the proposed energy-based method and contains criteria for performance evaluation and best-basis selection; section 4 presents the experimental results from analysis of a variety of underwater acoustic signals; section 5 discusses the implications of the results and conclusions; and section 6 presents suggestions for future work.

2. BACKGROUND

2.1 TRADITIONAL METHODS FOR SIGNAL COMPRESSION

The redundancy in signal/image data can be described in terms of correlation between data samples. The purpose of data compression is to remove such redundancy and to prepare the data for digital transmission or storage. The basic elements of a data compression system are depicted in figure 1. The first step is to process the signal by an operation that removes as much data correlation as possible. In the second step, these decorrelated data must be properly quantized; in the third step, the quantized samples are coded into a form suitable for transmission (coding may include such criteria as error detection or correction). Steps 2 and 3 are basically governed by the same considerations no matter what particular decorrelation scheme was is chosen in step 1. Signal decomposition and decorrelation are performed for data compression in the first step.

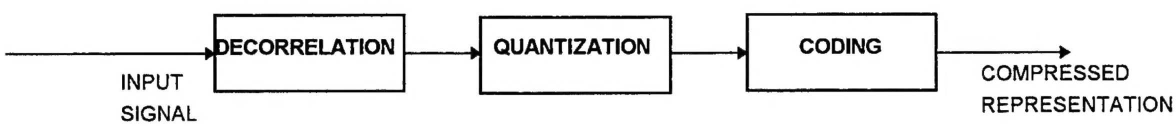

Figure 1. Block Diagram Components of a Data Compression System

One of the conventional data compression methods is the covariance or Karhunen-Loeve (K-L) transformation in which these data are decomposed into a set of uncorrelated components of decreasing statistical significance. Compression is achieved by selecting those components of greatest statistical importance and discarding the rest. Even though the K-L transformation provides perfect data decorrelation, it is not an efficient process; for applications with high data rates, the computational requirements are extremly expensive (Hunt, 1978).

The DFT has had a great impact on many applications of digital signal processing, including signal compression and reconstruction. Not only does the DFT provide data decorrelation, it also greatly reduces the computational requirements. A standard approach for analyzing a signal is to decompose that signal into a sum of simple building blocks. The DFT and DCT are the most well-known examples. The mathematical description of data compression with the DFT is

$$G(m,n) = \sum_{k=0}^{N-1} w(n,k) \sum_{j=0}^{N-1} g(j,k) w(m,j), \qquad (1)$$

where $g(\cdot)$ is the original signal (or image), $G(\cdot)$ is the transform of $g(\cdot)$, and $w(\cdot)$ is the kernel function. The kernel function for a transform compression scheme operating with the Fourier transform is

$$w(m, n) = \exp(2\pi jmn/N). \qquad (2)$$

Since the basis vector formed by the Fourier kernel function described in equation (2) is a cosine basis, it does not have compact support or finite energy. Thus, a large number of transform coefficients are required for containing a significant fraction of the total signal energy. In Fourier analysis, a signal *g(t)* is mapped into its frequency domain representation G(w), and hence, any time localization of the frequency information is lost. As mentioned in section 1, for low retention percentages (or high compression ratios), the performance of Fourier-based techniques rapidly deteriorate, and the reconstructed signal degrades severely. Wavelets and wavelet transforms have recently emerged as a useful alternative for many applications in signal processing. Because their basis functions have compact support and their transforms have good localization in both time and frequency domains, wavelets have opened up new avenues for improving signal compression methods (Chui, 1991).

2.2 WAVELET-BASED METHOD FOR SIGNAL COMPRESSION

In constrast to classical Fourier analysis, the time-frequency analysis of a signal is mapped into the time-frequency (*t-f*) plane using a suitable set of basis functions for the expansion. The uncertainty principle, a fundamental result in Fourier analysis, states that the time-bandwidth product of the signal satisfies the following inequality:

$$\Delta f \cdot \Delta t \geq \frac{1}{4\pi}, \tag{3}$$

where Δf and Δt are the spectral bandwidth and time scale of the signal (Rioul and Vetterli, 1991). It is desirable to construct basis functions, *wavelets*, with good localization properties in both time and frequency, subject to the uncertainty principle constraint. A wavelet expansion uses translations and dilations of one fixed function (or *mother wavelet*), $\psi(t) \in L^2(R)$. The mother wavelet $\psi(t)$ has to satisfy the *admissible condition*

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\psi(w)|^2}{w} dw < \infty, \tag{4}$$

where $\psi(w)$ is the Fourier transform of $\psi(t)$. In a continuous wavelet transform, the translation and dilation parameters vary continuously. The wavelet transform of a signal $g(t)$ is now defined as

$$W(a, b) = \langle g, \psi_{a,b} \rangle, \tag{5}$$

where

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \text{ with } a, b \in R,\ a \neq 0. \tag{6}$$

By a wavelet decomposition of a given function $g(t)$ one can represent that function as

$$g(t) = \sum_{n} \sum_{k} c_{nk} \ \psi_{nk}(t), \tag{7}$$

where n and k range over Z and Z^2, respectively, and the c_{nk} are the coefficients. Each of the functions $\psi_{nk}(t)$ belongs to one of a finite number of families $\{\psi_{nk}(t)\}$, and the parameters n and k are related to the scale and location of this function.

One way to apply wavelet decompositions for data compression is to approximate $g(t)$ by a finite sum of functions $\psi_{nk}(t)$. Because the values of the functions $\psi_{nk}(t)$ stay the same, the information content of the signal g is captured in the coefficients c_{nk}. The signal $g(t)$ is compressed by (1) selecting the largest coefficients, and (2) applying traditional coding techniques to the sequence of selected coefficients.

The most common method of selecting wavelet coefficients of a signal is called the *global threshold* method (Nacken, 1993). This technique selects the wavelet coefficients based on a universal threshold level. Because errors introduced by compression depend only on the size of the coefficients, one can eliminate the smallest coefficients and still have a reasonably good approximation of the original signal. However, when large compression ratios and high-quality reconstruction are required, the performance of the global threshold technique is inadequate. In many undersea applications, such as sidescan sonar images and underwater acoustic signals, noise and other interference contribute a great deal to degrading signals and images. The need for a new technique that can improve the performance of data compression thus becomes important.

3. PROPOSED METHOD

3.1 ENERGY-BASED SELECTION OF WAVELET COEFFICIENTS

3.1.1 Motivation

This report proposes a new energy-based method for selection of wavelet coefficients for signal compression that was motivated by examination of the tree structure of the wavelet decomposition (see figure 2). The tree structure as shown consists of several resolution levels, where resolution level k represents an approximation of the original signal with a resolution of one point for every 2^k points of the original signal (Mallat, 1989). For the signals analyzed, the wavelet coefficients with the largest magnitudes occurred among only a few levels of the decomposition tree. Physically, this finding means that details of the original signal from only those resolution levels are retained; whereas the signal itself has several levels in its decomposition tree.

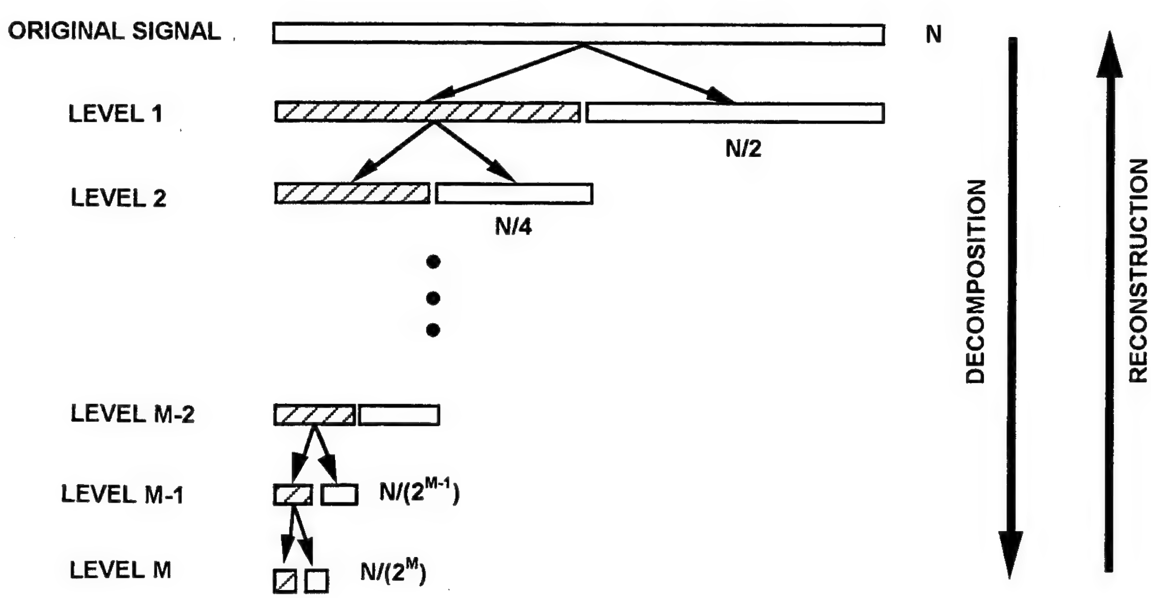

Figure 2. Conceptual Schematic of Wavelet Decomposition Tree

3.1.2 Algorithm

The energy-based technique is based on the *mean energy* contained at each level of the wavelet decomposition tree. The number of wavelet coefficients selected from a particular level is proportional to the mean energy contained at that level. The largest wavelet coefficients from each level are selected in proportion to the mean-energy content of that level. The baseline algorithm is shown in the following outlined area.

- Let the length of the signal (number of samples) $N = 2^M$
  Maximum number of signal resolution levels $= M$
- At level k of the decomposition, $k = 1, 2, \ldots, M$
  Number of wavelet coefficients available is $N_k = \dfrac{N}{2^k}$
- Let the wavelet coefficients at level k be $\{c_{kj}\}$, $j = 1, 2, \ldots, N_k$
  The mean energy of level k is defined as $\overline{E}_k = \dfrac{1}{N_k}\sum_{j=1}^{N_k} c_{kj}^2$
- Number of coefficients desired from level k, $\widetilde{N}_k = \dfrac{\overline{E}_k}{\sum \overline{E}_k} \times \dfrac{\text{Percent Retention}}{100} \times N$

In the typical situation, the number of coefficients desired is less than or equal to the number of coefficients available; that is, $\widetilde{N}_k \leq N_k$. However, a special case that occurs frequently in practice is $\widetilde{N}_k > N_k$; that is, the number of coefficients desired from level k is greater than the number available at that level. This situation is handled by selecting additional coefficients from other levels to make up the difference. Coefficients are allocated based on the energy-ranking scheme. In this scheme, the first choice is the level with the maximum mean energy, the next choice is the level with the second-highest mean energy, etc. (The enhanced algorithm is shown in the outlined area following this paragraph.)

- For levels $k = 1, 2, \ldots, K$

  If $\tilde{N}_k > N_k$

  Number of coefficients selected from level k, $\hat{N}_k = N_k$

  else

  Number of coefficients selected from level k, $\hat{N}_k = \tilde{N}_k$

  end if

  Difference between coefficients desired and selected at level k, $D_k = \tilde{N}_k - \hat{N}_k$

end for

Total additional coefficients desired, $D = \sum_{k=1}^{K} D_k$

- While $D > 0$

  Let level k_M have maximum mean energy; i.e., $\bar{E}_{k_M} = \max\{\bar{E}_k\}, k = 1, 2, \ldots, K$

  Let number of coefficients desired from level k_M be $\tilde{N}_{k_M}$

  Let number of coefficients available at level k_M be N_{k_M}

  If $\tilde{N}_{k_M} + D < N_{k_M}$

  Number of coefficients selected from level k_M, $\hat{N}_{k_M} = \tilde{N}_{k_M} + D$

  else

  Number of coefficients selected from level k_M, $\hat{N}_{k_M} = N_{k_M}$

  end if

  Update total additional coefficients desired, $D = D - \left(\hat{N}_{k_M} - \tilde{N}_{k_M}\right)$

  Eliminate level k_M from further consideration

end while

This selection method ensures that if the mean energy at a given decomposition level is sufficiently large, some wavelet coefficients from that level will be selected in the compressed representation of that signal. This result is in contrast to the global threshold technique, wherein the largest coefficients in a universal sense are retained independent of their distribution across levels. In general, it was observed that the energy-based selection method retains wavelet coefficients across a broader spectrum of levels in the decomposition tree. Two illustrative analyses for porpoise and dolphin acoustic signals are presented in paragaph 3.1.3.

3.1.3 Example

For the dolphin and porpoise signals shown in figure 3, the distribution of selected wavelet coefficients across the various resolution levels is listed in tables 1 and 2 for different retention percentages. For example, a 10-percent retention of the porpoise signal using the conventional global thresholding method results in selection of coefficients from levels 1 through 4. In contrast, the energy-based method results in selection of coefficients from levels 1 through 7. In a physical sense, this result means that details of the original signal are utilized in the reconstruction in the energy-based method across a wider range of resolution levels than in the global-thresholding method. The resulting enhancement in the signal reconstruction with the energy-based method is shown in figure 4. The reconstructed signal obtained from energy-based compression is compared to the reconstruction from global thresholding for the porpoise sound (using 10-percent retention of the Daubechies four-wavelet basis). The former retains significantly finer detail of the original signal than the latter and is illustrated in figure 5 by zooming in on a segment of the porpoise signal reconstruction. A tradeoff occurs in the large-amplitude regions of the reconstructed signal. Since global thresholding retains the largest coefficients overall, some of the peak amplitudes are reproduced more accurately than for energy-based reconstruction. This effect does not represent significant information loss in the energy-based signal reconstruction because the overall trends are still retained. However, presence of some signal detail results in improved interpretation, such as feature extraction and classification.

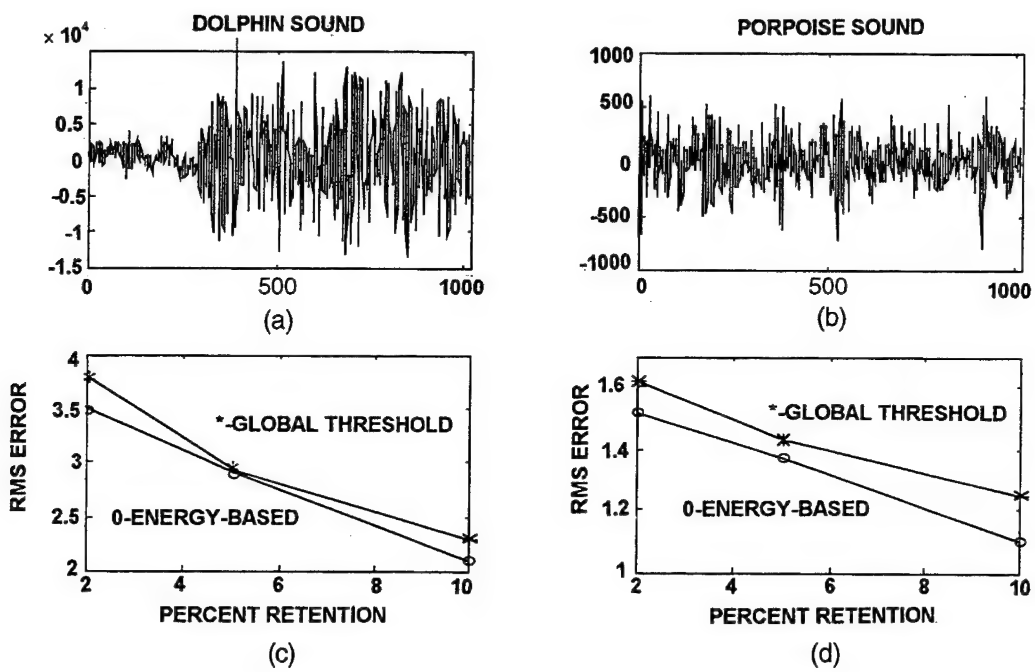

Figure 3. (a) and (b): Time Series of Dolphin and Porpoise Sounds; (c) and (d): Root-Mean-Square-Error Comparisons for Reconstruction of Dolphin and Porpoise Signals Using the Daubechies Four-Wavelet Basis for Decomposition

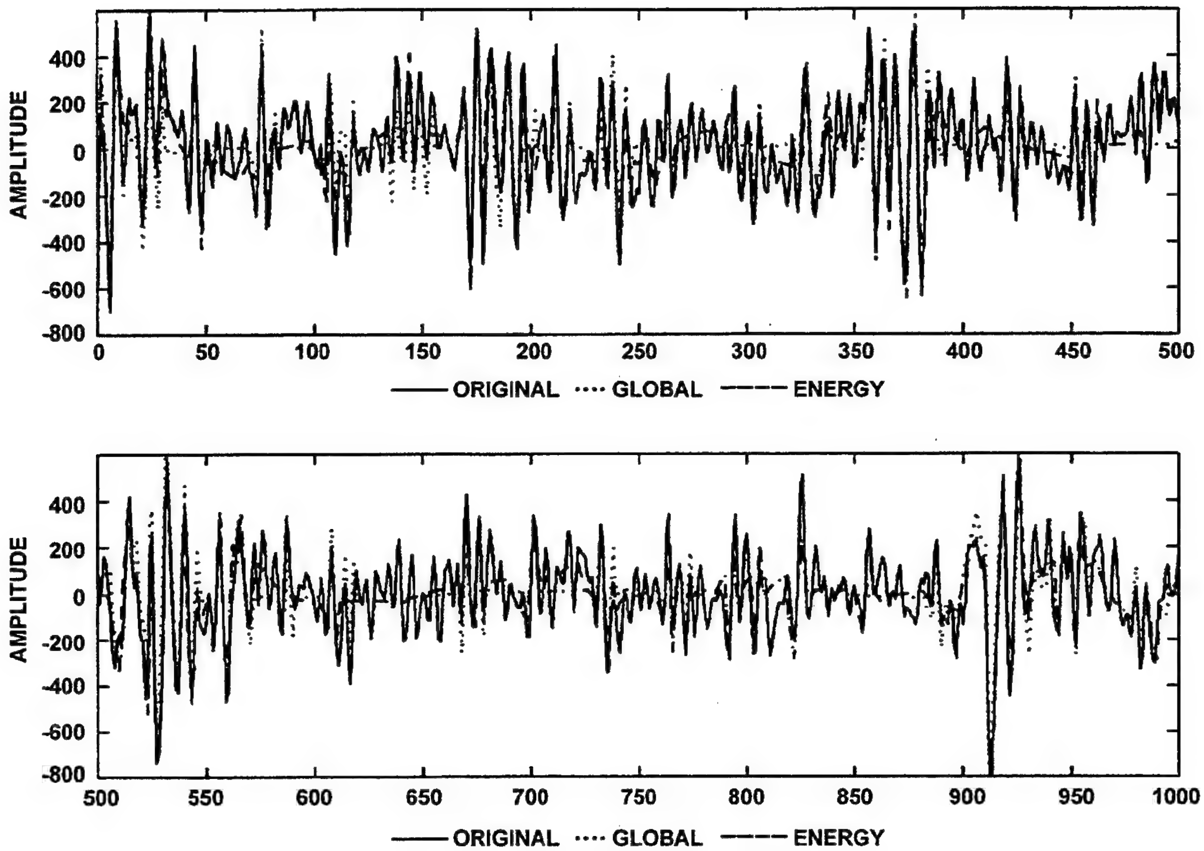

Figure 4. Signal Reconstruction: Porpoise Sound with 10-Percent Retention Using Daubechies Four-Wavelet Basis

Tables 1 and 2 show the distribution of wavelet coefficients across resolution levels for varying retention percentages **(% Ret)** using global thresholding **(G)** and mean energy **(E)** selection methods. Signals are analyzed using the Daubechies four-wavelet basis. The performance index of reconstruction is the signal-to-error ratio (SER) in decibels and has been calibrated for equivalent numbers of coefficients selected using the two methods.

Table 1. Distribution of Selected Wavelet Coefficients for Porpoise Signal

% Ret	**2**		**5**		**10**		**20**		**40**	
Level	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**
1	1	2	4	6	19	11	78	23	178	45
2	12	8	35	21	64	41	97	83	158	175
3	6	6	11	14	18	28	26	57	52	115
4	1	1	1	4	1	7	1	14	12	28
5		1		3		6	1	12	7	24
6		1		3		6	2	12	3	16
7				1		2		3		6
8										1
9										
10										
Total	*20*	*19*	*51*	*52*	*102*	*101*	*205*	*204*	*410*	*410*
SER	**1.60**	**1.69**	**2.73**	**3.02**	**3.95**	**4.82**	**5.84**	**7.56**	**9.54**	**11.59**

Table 2. Distribution of Selected Wavelet Coefficients for Dolphin Signal

% Ret	**2**		**5**		**10**		**20**		**40**	
Level	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**
1		1		2	3	5	39	10	156	19
2	9	6	23	15	56	30	101	61	150	185
3	11	9	25	23	35	46	47	93	72	128
4		3	3	8	8	17	18	34	30	64
5				1		1		3	2	5
6				0		1		1		2
7				1		1		2		4
8						1		1		2
9										
10										
Total	*20*	*19*	*51*	*50*	*102*	*102*	*205*	*205*	*410*	*409*
SER	**1.63**	**1.78**	**3.67**	**3.73**	**5.78**	**6.31**	**8.97**	**9.17**	**13.16**	**13.77**

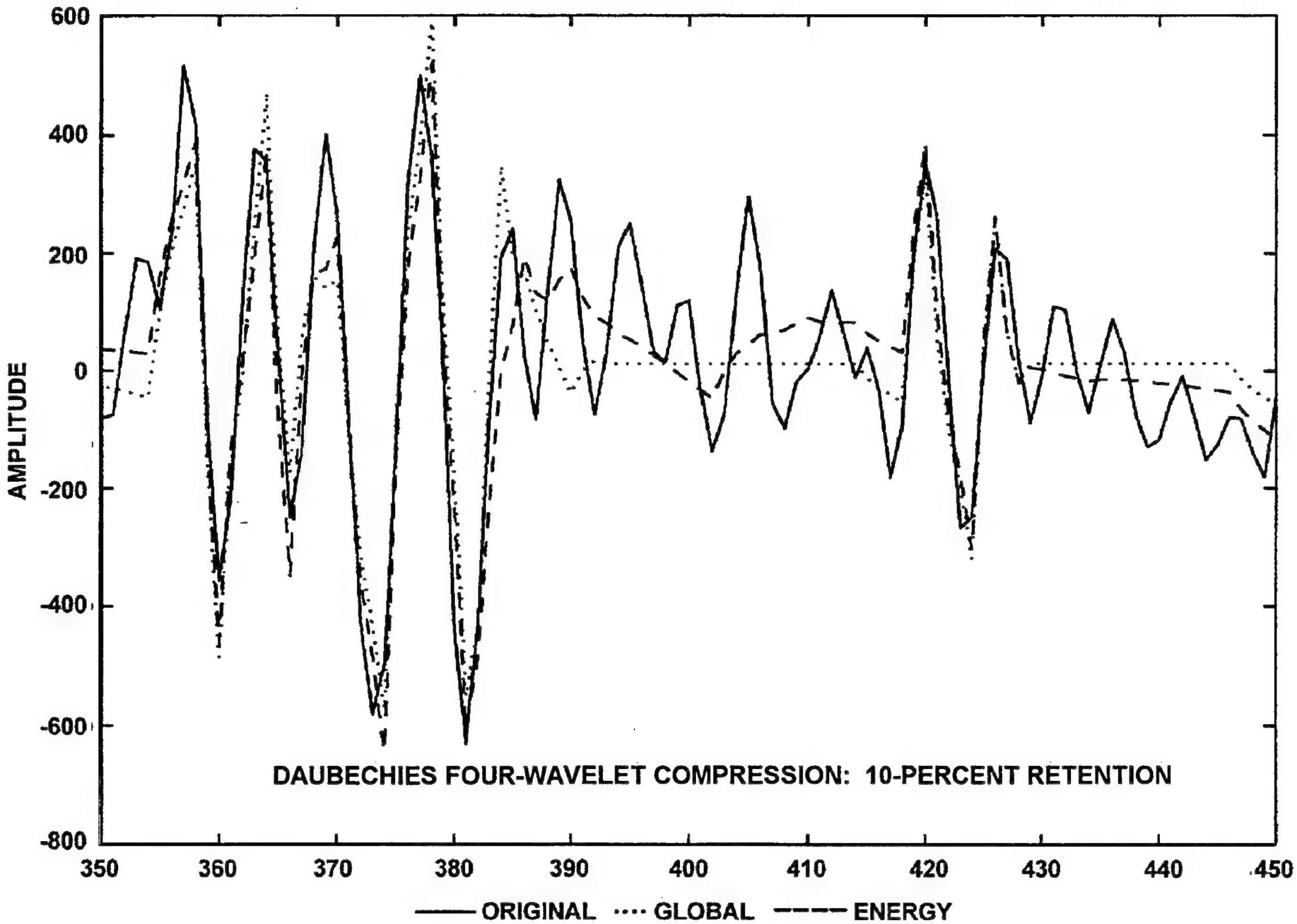

Figure 5. Porpoise Sound Reconstruction: Time Segment from 350-450

3.2 PERFORMANCE CALIBRATION

The performance index for signal reconstruction is defined with respect to the fidelity of the reconstructed signal *r(t)* from its compressed representation. The measure of "goodness" is how closely this reproduction matches the original signal *s(t)* in terms of the reconstruction error, *e(t) = s(t) - r(t),* and is the SER,

$$\text{SER} = 20\log\left[\frac{S_{rms}}{E_{rms}}\right] \text{ (in units of dB),} \tag{8}$$

where S_{rms} and E_{rms} are the rms values of signal *s(t)* and error *e(t)* , respectively. The SER criterion is analogous to the well-known signal-to-noise ratio (SNR) and shares the property that signal amplitude levels are factored into the measure.

An auxiliary effect that occurs in the energy-based method is the presence of integer-number rounding for wavelet coefficient selection among the different resolution levels. This effect is demonstrated, for example, in table 1 at the 2-percent retention level for the porpoise signal. The total number of coefficients selected over the different levels using the energy technique is 19, whereas on an absolute basis 2 percent of this 1024-length signal leads to 20 coefficients (which is the number selected by global thresholding). To accurately compare performance among the two coefficient selection methods, the SER index is calibrated to predict performance assuming that an identical number of coefficients were chosen using the energy method (in this case, 20). The performance calibration algorithm uses a quadratic curve-fit to interpolate the SER for a new number of wavelet coefficients and is shown in the following outlined area.

- Let total number of wavelet coefficients $= N$,
  and let number of wavelet coefficients selected $= n$
- Let number of coefficients selected by global thresholding $= N_g$,
  and let number of wavelet coefficients selected by mean energy $= N_e$
- Assuming that signal reconstruction error $e(n) = a \cdot n^2 + b \cdot n + c$, known points are
  (1) $n=0,\ e(0) = c \cong S_{rms}$
  (2) $n=N_e,\ e(N_e) = a \cdot N_e^2 + b \cdot N_e + c \cong E_e$
  (3) $n=N,\ e(N) = a \cdot N^2 + b \cdot N + c \cong 0$
- Solving for *a, b, c* leads to
  $$a = \frac{N_e S_{rms} + N(E_e - S_{rms})}{N_e^2 N - N^2 N_e}$$
  $$b = -\frac{N_e^2 S_{rms} + N^2(E_e - S_{rms})}{N_e^2 N - N^2 N_e}$$
  $$c = S_{rms}$$
- Hence, for $n=N_g,\ e(N_g) = a \cdot N_g^2 + b \cdot N_g + c \cong E_g$

This performance calibration is required in about 60 percent of the cases involving a mean energy selection-based comparison with global thresholding.

3.3 ENTROPY CONSIDERATIONS FOR "BEST-BASIS" SELECTION

An open issue in any wavelet decomposition is the choice of the basis function or resolution kernel. Here, the approach of Coifman and Wickerhauser (1992) is adopted in selecting the best basis. The optimal basis is defined as the basis function that results in the wavelet decomposition with minimum entropy. The entropy of the wavelet decomposition $\{w_i\}_{i=1}^{N}$ of the signal $\{s_i\}_{i=1}^{N}$ is

$$E = \sum_{i=1}^{N} w_i^2 \log \frac{1}{w_i^2} \; ; \tag{9}$$

thus, $E_m = \min\{E_j\}$ where basis family j=1, 2, . . ., .

In a thermodynamic sense, entropy is regarded as the measure of chaos in a physical system (Resnick and Halliday, 1975) and can be thought of as the amount of "scatter" in the wavelet coefficients of the signal. Hence, selection of the basis with minimum entropy is equivalent to picking the kernel function that results in least variability in the wavelet coefficients.

4. EXPERIMENTAL RESULTS

Experimental tests were done using both the global threshold and the energy-based wavelet coefficient selection technique for signal compression and reconstruction of four sets of acoustic data. This analysis was performed using the Wavelet Toolkit Software as described in the appendix. Two data sets were from biological sources, and two were from undersea vehicles. The SER and the entropy levels were recorded for several Daubechies-basis functions (Daubechies, 1992) using 2-, 5-, 10-, 20-, and 40- percent retention levels. These results are displayed and referenced in the following sections.

4.1 ANALYSIS OF TWO BIOLOGICAL SIGNALS

4.1.1 Porpoise Signal

The porpoise signal shown in figure 3(a) was analyzed using conventional global thresholding and the proposed energy-based method. The experimental results from each method are shown in table 3 where **G** represents global thresholding, **E** represents the mean-energy wavelet coefficient technique. The SER is measured in decibels for 2-, 5-, 10-, 20-, and 40-percent retention levels.

Table 3. Signal-to-Error Ratio and Entropy Values for the Porpoise Signal

% Ret	**2**		**5**		**10**		**20**		**40**		**Entropy**
Basis	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**($x10^7$)**
1 SER	1.09	1.09	2.07	2.41	3.28	4.02	4.90	6.76	8.41	10.85	3.75
2 SER	1.60	1.69	2.73	3.02	3.95	4.82	5.84	7.56	9.54	11.59	3.21
3 SER	1.66	1.72	2.74	3.24	4.13	4.96	6.17	7.92	10.57	11.90	3.04
4 SER	1.56	1.68	2.87	3.18	4.37	5.17	6.89	8.29	10.47	12.49	3.27
5 SER	1.70	1.94	3.08	3.51	4.64	5.53	7.41	8.81	12.44	13.51	2.94

Performance comparisons between the energy-based technique and the global threshold method for signal reconstruction of the porpoise sound are displayed for Daubechies bases 4, 6, 8, and 12 in figure 6 (a) through (d), respectively. When plotted against the percent-retention levels, the energy-based technique outperforms the global threshold method for the porpoise signal at each Daubechies basis.

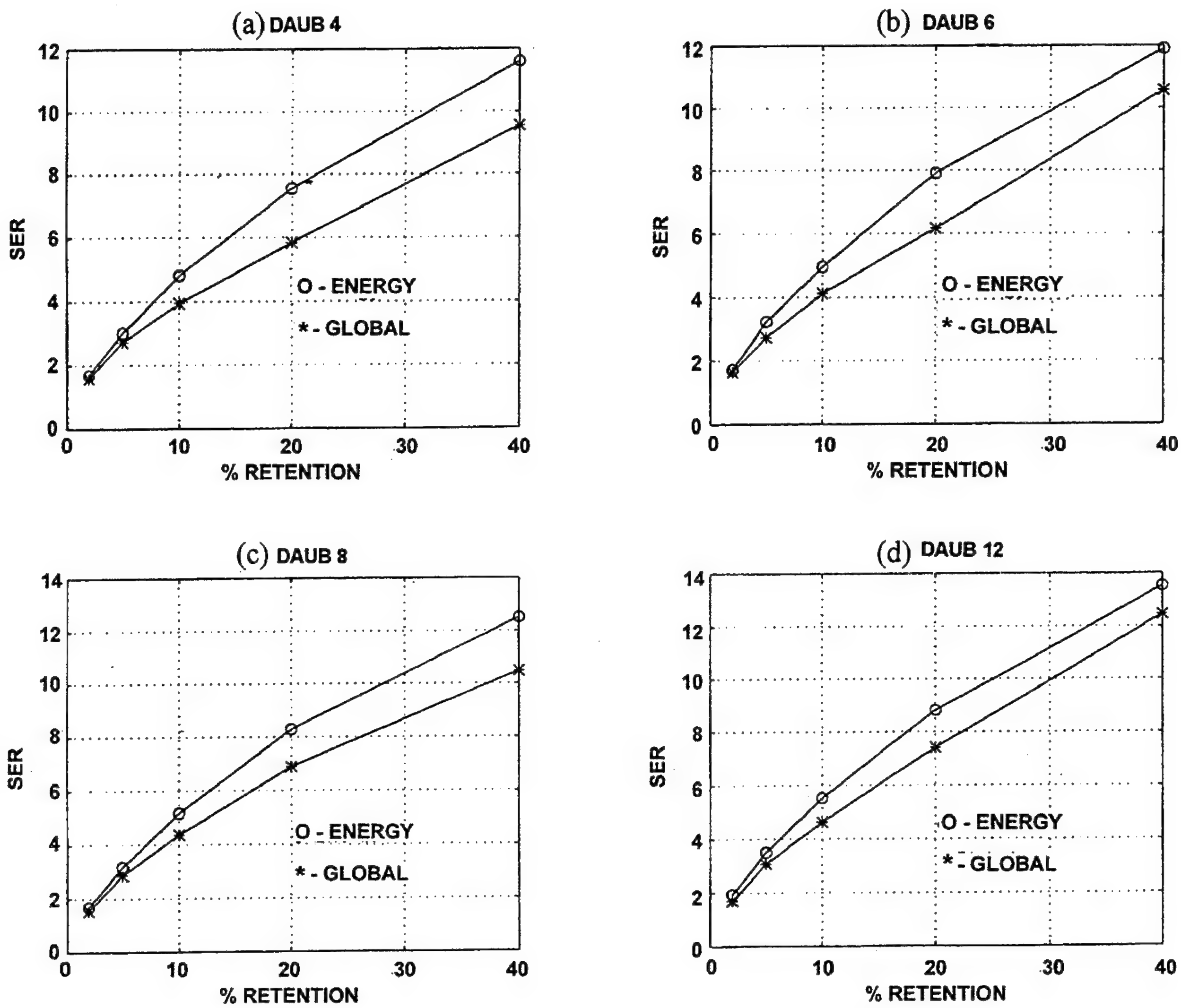

Figure 6. Reconstruction Performance Comparison for the Porpoise Signal

Figure 7 displays the entropy values and reconstruction performance for the porpoise signal. The basis with the minimum entropy is Daubechies 12 as shown in figure 7 (a) and (b). This basis was observed to provide the best performance for signal reconstruction.

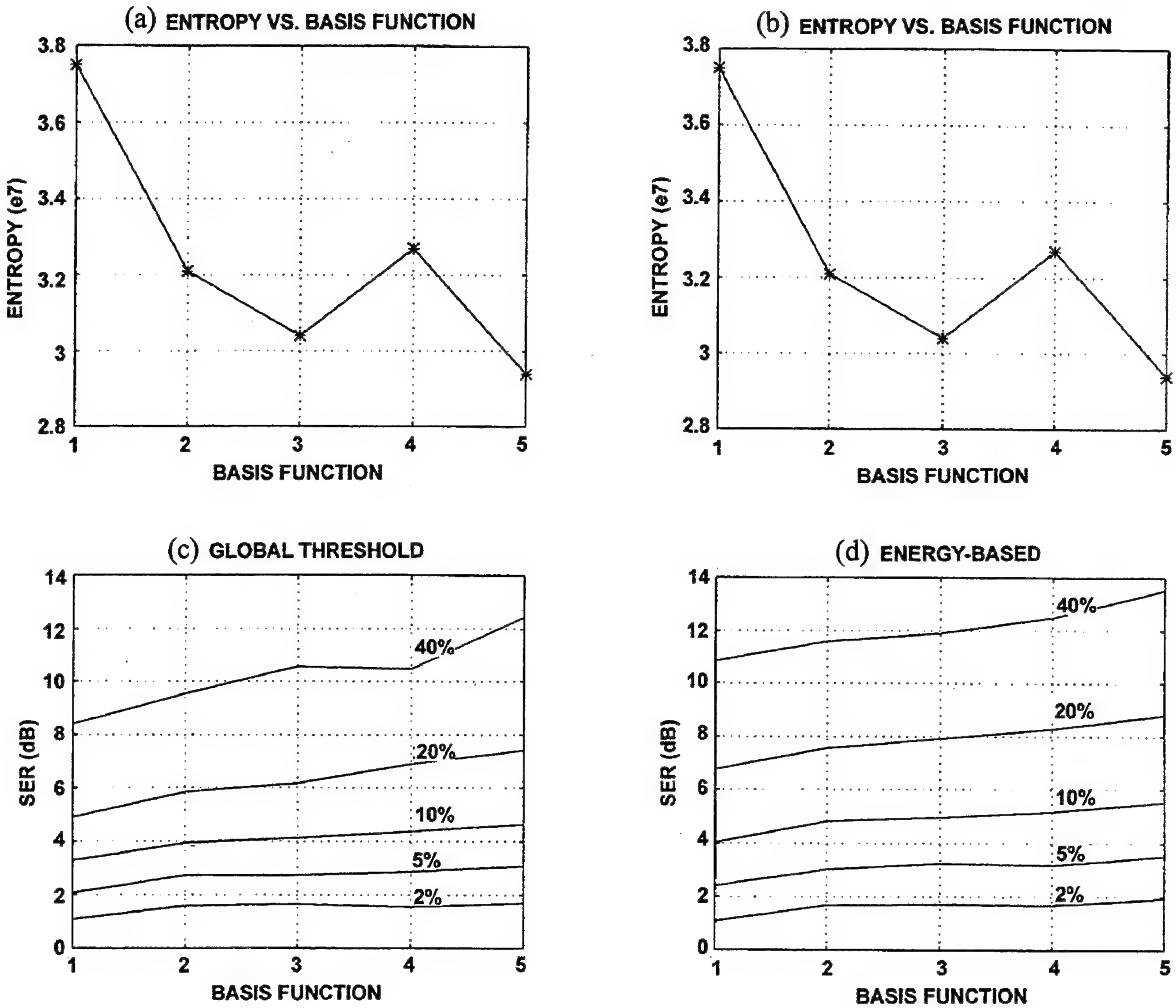

Figure 7. Reconstruction Performance of the Porpoise Sound

The reconstruction performance of the porpoise sound using the global threshold method at selected percent-retention levels is displayed in figure 7 (c). The SER was observed to increase with increasing percent-retention levels. The best performance for signal reconstruction in terms of the maximum SER occurs for Daubechies 12, as predicted by the minimum entropy point. Similarly, the reconstruction performance for the porpoise sound using the energy-based method for the same retention levels is displayed in figure 7 (d) and shows the same basic characteristics observed in figure 7 (c).

4.1.2 Dolphin Signal

Similar testing and analysis were performed using the dolphin signal shown in figure 3 (b). The results from the global threshold and the energy-based methods are shown in table 4 where **G** represents global thresholding, **E** represents the mean-energy wavelet coefficient technique. The SER is measured in decibels for 2-, 5-, 10-, 20-, and 40-percent retention levels.

Table 4. Signal-to-Error Ratio and Entropy Values for the Dolphin Signal

% Ret	**2**		**5**		**10**		**20**		**40**		**Entropy**
Basis	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**($x10^7$)**
1 SER	1.61	1.76	2.85	3.07	4.42	5.26	6.94	7.97	10.87	11.76	2.59
2 SER	1.63	1.78	3.67	3.73	5.78	6.31	8.97	9.17	13.16	13.77	2.34
3 SER	2.36	2.25	4.17	4.50	6.57	7.01	10.36	9.93	16.12	15.47	2.11
4 SER	2.18	2.27	4.30	4.54	6.90	6.99	11.44	10.35	16.76	16.25	2.13
5 SER	2.45	2.49	4.04	4.68	7.43	7.55	12.55	11.09	18.79	17.97	2.09

Performance comparisons were done to further analyze the signal reconstruction of the dolphin sound using the global threshold and energy-based methods for Daubechies 4, 6, 8, and 12. Figure 8 shows that a crossover occurs in the reconstruction performance of the global threshold and the energy-based methods for Daubechies bases 6, 8, and 12. Close analysis of the data presented in table 2 verifies that no crossover occurs for Daubechies 4 in figure 8(a) (that is, the energy-based technique outperforms the global threshold technique at all retention levels). The crossover points for the other three basis functions (figure 8 (b) through (d)) occur somewhere between 10-and 20-percent retention.

Table 5 shows that the wavelet coefficient selection distribution for the porpoise signal using Daubechies 12 progresses more deeply into the decomposition tree for the energy-based method than for the global threshold. The SER results further verify that no crossover occurred in the reconstruction of the porpoise signal. The distribution of the wavelet coefficients for the dolphin signal using Daubechies 12 as shown in table 6, however, appears to converge to the same level of the decomposition tree for both methods. It is clear from analyzing the SER values that a crossover in reconstruction performance occurs between 10-and 20-percent retention (that is, the SER values for the global method exceed those for the energy-based method).

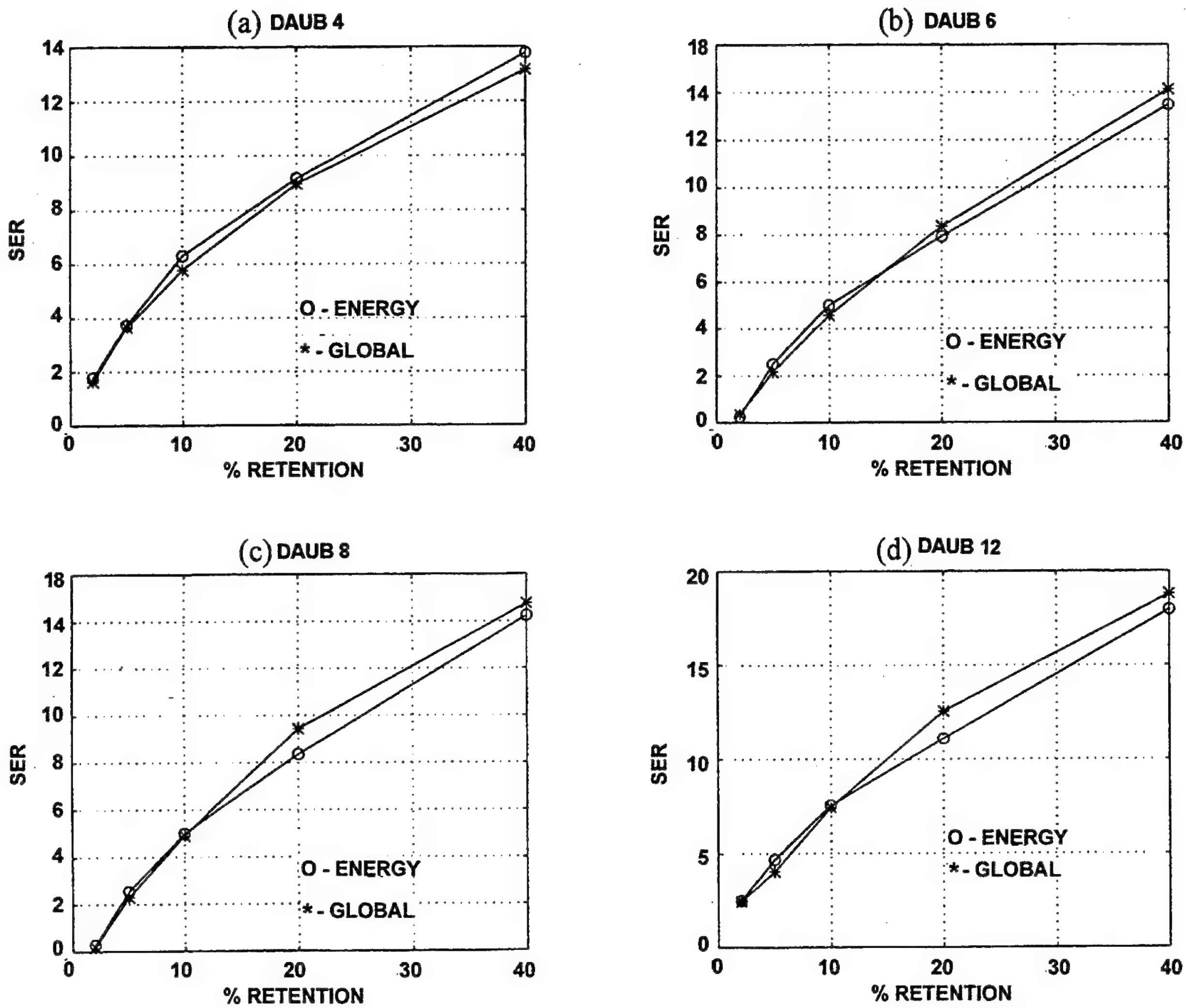

Figure 8. Reconstruction Performance Comparison for the Dolphin Signal

Table 5. Distribution of Selected Wavelet Coefficients for Porpoise Signal

% Ret	**2**		**5**		**10**		**20**		**40**	
Level	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**
1	1	1	1	3	3	6	30	12	143	25
2	14	9	39	23	77	45	127	91	171	191
3	5	6	10	16	20	31	37	63	65	126
4		2	1	4	2	8	7	17	20	33
5		1		2		5	3	9	7	18
6		1		3		6	1	13	4	16
7										1
8										
9										
10										
Total	*20*	*20*	*51*	*51*	*102*	*101*	*205*	*205*	*410*	*410*
SER	**1.70**	**1.94**	**3.08**	**3.51**	**4.64**	**5.53**	**7.41**	**8.81**	**12.44**	**13.51**

Table 6. Distribution of Selected Wavelet Coefficients for Dolphin Signal

% Ret	**2**		**5**		**10**		**20**		**40**	
Level	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**
1				1		1	8	3	109	5
2	5	5	26	13	50	26	110	53	174	209
3	15	11	24	29	44	58	66	116	88	128
4		3	1	8	8	15	21	31	35	62
5						1		1	2	3
6						1		1	2	2
7										
8										
9										
10										
Total	*20*	*19*	*51*	*51*	*102*	*102*	*205*	*205*	*410*	*409*
SER	**2.45**	**2.49**	**4.04**	**4.68**	**7.43**	**7.55**	**12.55**	**11.09**	**18.79**	**17.99**

Tables 5 and 6 show the wavelet coefficient selection distribution across decomposition levels for varying retention percentages **(% Ret)** using global thresholding (G) and mean-energy (E) selection methods. Signals were analyzed using the Daubechies 12-wavelet basis. The performance index of reconstruction is the SER in decibels and has been calibrated for equivalent numbers of coefficients selected using the two methods.

Entropy values and reconstruction performance for the dolphin signal are shown in figure 9. The minimum entropy for the dolphin signal occurs at Daubechies 12; therefore, the best performance for signal reconstruction is expected to occur at this basis as was observed with the porpoise signal.

Figures 9 (c) and (d) show the reconstruction performance of the dolphin sound using the global threshold and energy-based methods respectively. The SER for the dolphin signal reconstruction again increases with increasing percent-retention levels. The best performance for signal reconstruction in terms of the maximum SER occurs for Daubechies 12, as was previously seen with the porpoise signal.

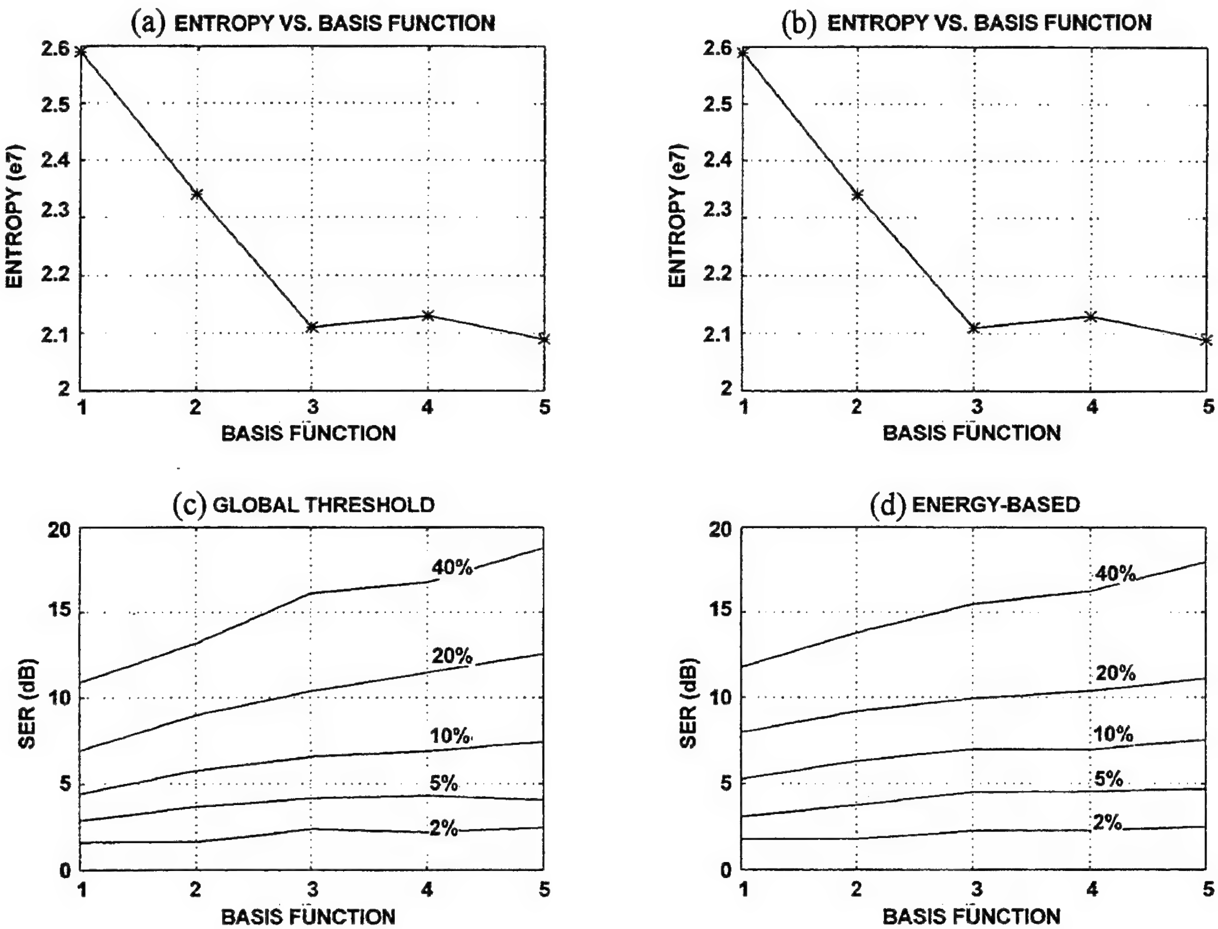

Figure 9. Reconstruction Performance of the Dolphin Sound

4.2 ANALYSIS OF SIGNALS FROM TWO UNDERSEA VEHICLES

4.2.1 Undersea Vehicle A

Signal data from vehicle A were also analyzed using conventional global thresholding and the proposed energy-based method. The experimental results from each method are shown in table 7.

Table 7. Signal-to-Error Ratio and Entropy Values for Vehicle-A Signal

% Ret	2		5		10		20		40		Entropy
Basis	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**($x10^7$)**
1 SER	0.28	0.46	0.68	0.46	1.40	1.84	2.82	3.57	5.99	7.63	4.73
2 SER	0.44	0.44	1.03	1.04	1.95	2.00	3.62	4.01	6.90	8.53	5.88
3 SER	0.42	0.45	0.97	1.04	1.81	1.98	3.34	3.91	6.64	8.36	5.59
4 SER	0.31	0.43	0.74	0.98	1.43	1.84	2.82	3.59	6.00	7.58	4.79
5 SER	0.54	0.60	1.13	1.32	1.98	2.40	3.58	4.44	6.82	8.46	4.24

Table 7 shows experimental results of the vehicle-A signal reconstruction using global thresholding (**G**) and mean-energy (**E**) wavelet coefficient selection techniques. The SER is measured in decibels for 2-, 5-, 10-, 20-, and 40- percent retention levels.

Performance comparisons between the energy based technique and the global threshold method for signal reconstruction of the sound from vehicle A are displayed for Daubechies bases 4, 6, 8, and 12 in figure 10 (a) through (d), respectively. When plotted against the percent-retention levels, the energy-based technique outperforms the global threshold method for the signal at each Daubechies basis with a noticeable improvement as percent-retention increases.

Figure 11 (a) and (b) display the entropy values for vehicle A. The basis with the minimum entropy is again Daubechies 12. The reconstruction performance for the vehicle sound using global thresholding and the energy-based method for the same retention levels are displayed in figure 11 (c) and (d), respectively. The SER values were observed to increase slightly with increasing retention levels for both wavelet selection coefficient methods.

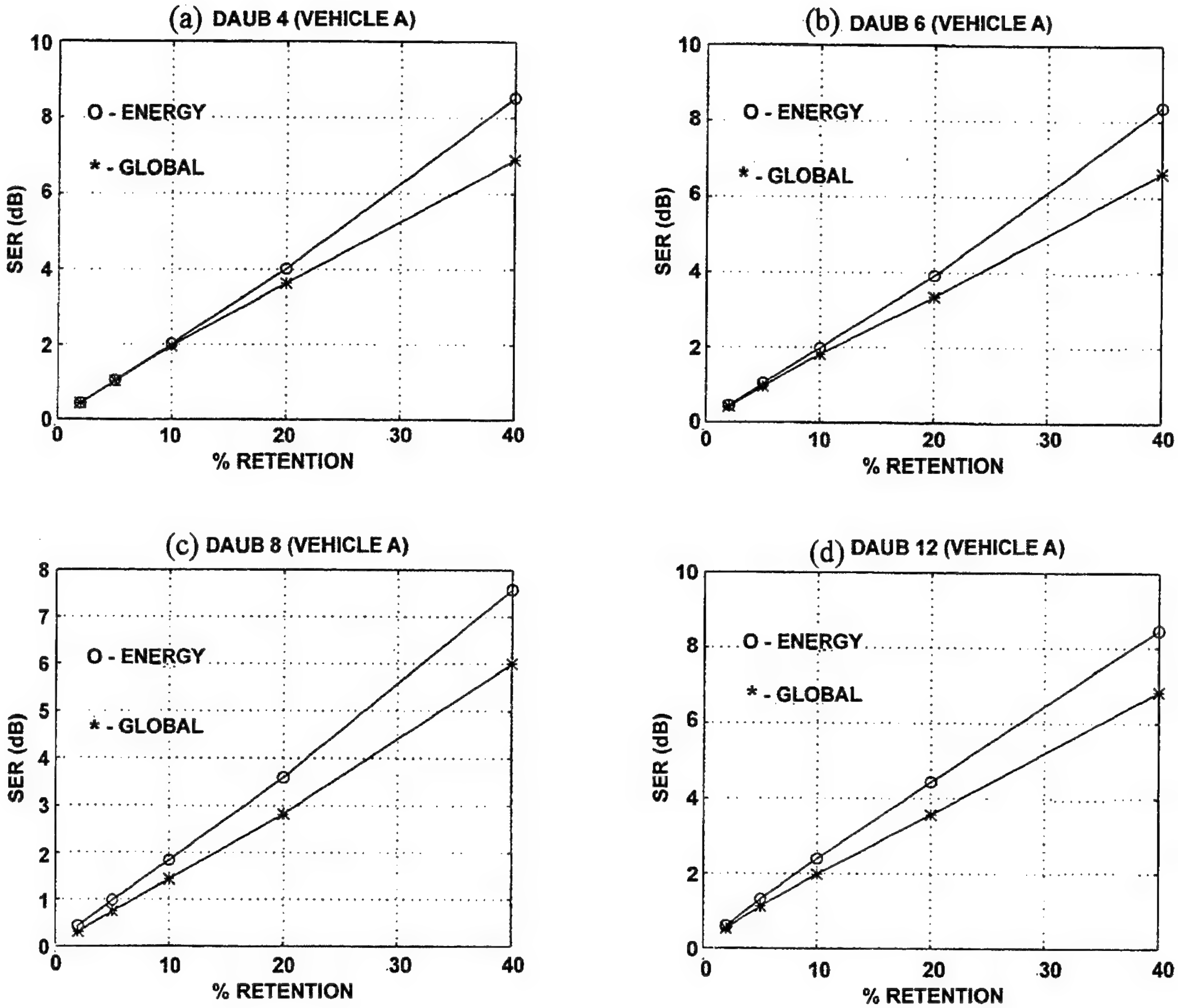

Figure 10. Reconstruction Performance Comparison for Vehicle A

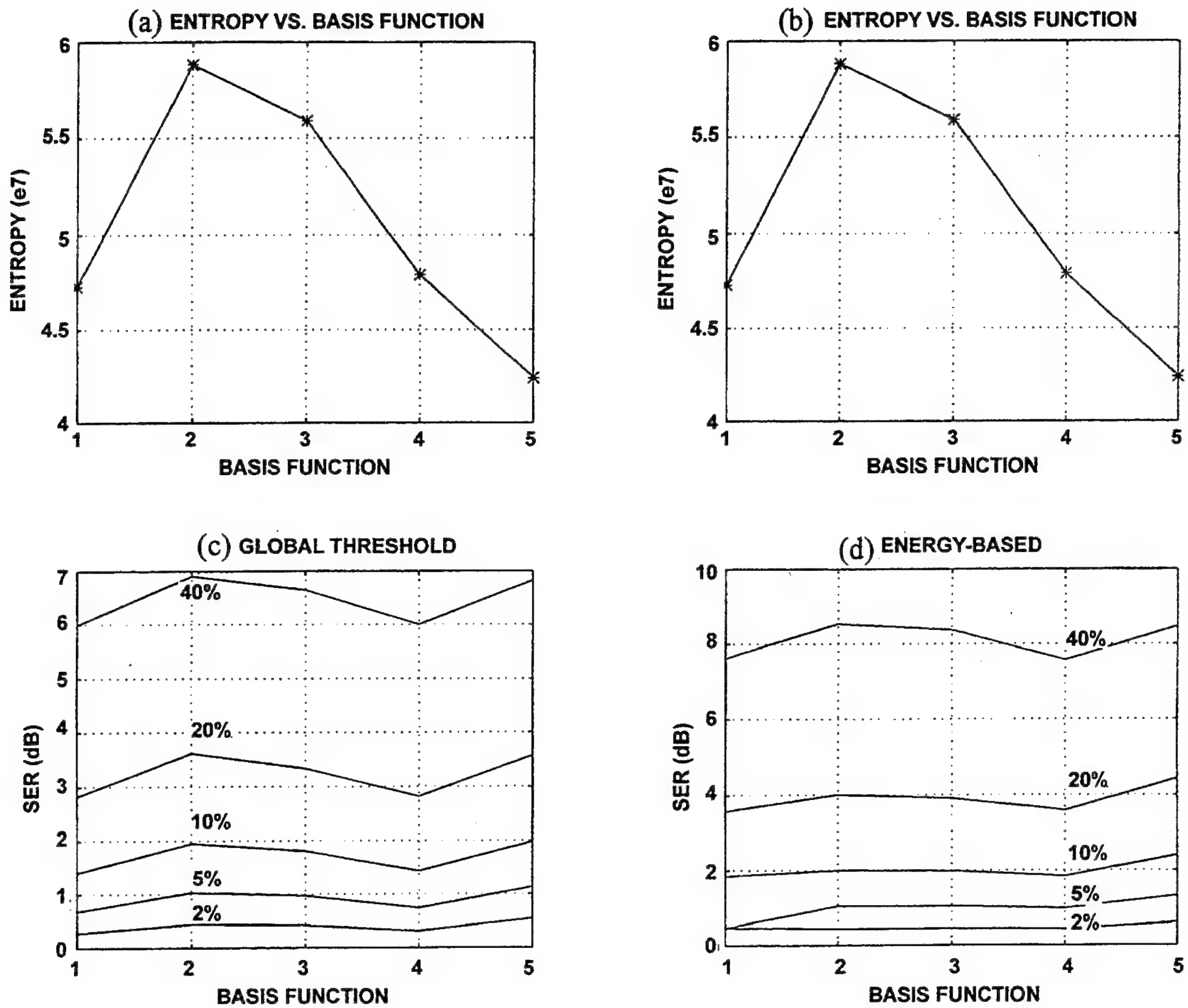

Figure 11. Reconstruction Performance of Vehicle-A Sound

4.2.2 Undersea Vehicle B

Similar testing and analysis were done using data recorded from a second source which will be referred to as "undersea vehicle B." The results from the global threshold and the energy-based methods are shown in table 8.

Table 8. Signal-to-Error Ratio and Entropy Values for Vehicle-B Signal

% Ret	**2**		**5**		**10**		**20**		**40**		**Entropy**
Basis	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**($x10^6$)**
1 SER	2.72	2.67	3.44	3.44	4.38	4.49	6.18	6.52	9.70	10.92	3.32
2 SER	2.77	2.78	3.63	3.63	4.91	4.90	7.19	7.41	10.79	12.60	3.70
3 SER	2.76	2.74	3.60	3.56	4.81	4.78	6.98	7.17	10.55	12.15	3.61
4 SER	2.68	2.66	3.38	3.35	4.38	4.39	6.19	6.41	9.57	10.81	3.32
5 SER	2.57	2.60	3.14	3.20	4.01	4.12	5.61	5.89	8.82	9.92	3.07

Table 8 shows the experimental results of the dolphin sound reconstruction using global thresholding (**G**) and mean-energy (**E**) wavelet coefficient selection techniques. The SER is measured in decibels for 2-, 5-, 10-, 20-, and 40- percent retention levels.

Performance comparisons were done to analyze the signal reconstruction of vehicle B using the global threshold and energy-based methods for Daubechies 4, 6, 8, and 12. Figure 12 shows that both methods perform similarly at lower (2-percent to 10-percent) retention levels, but begin to diverge significantly at higher (20-percent to 40-percent) retention levels with the energy-based method outperforming global thresholding.

Table 9 shows that the wavelet coefficient selection distribution for the vehicle-A signal using Daubechies 12 progresses more deeply into the decomposition tree for the energy-based method than for global thresholding. The SER results further verify that no crossover occurred in the reconstruction of the vehicle-A signal. The distribution of the wavelet coefficients for vehicle B using Daubechies 12 is shown in table 10. As seen with vehicle A, the wavelet coefficient distribution for vehicle B also spans across more levels when using the energy-based technique. These results suggest a direct correlation exists between the reconstruction performance using the energy-based method and the coefficient distribution in the decomposition tree.

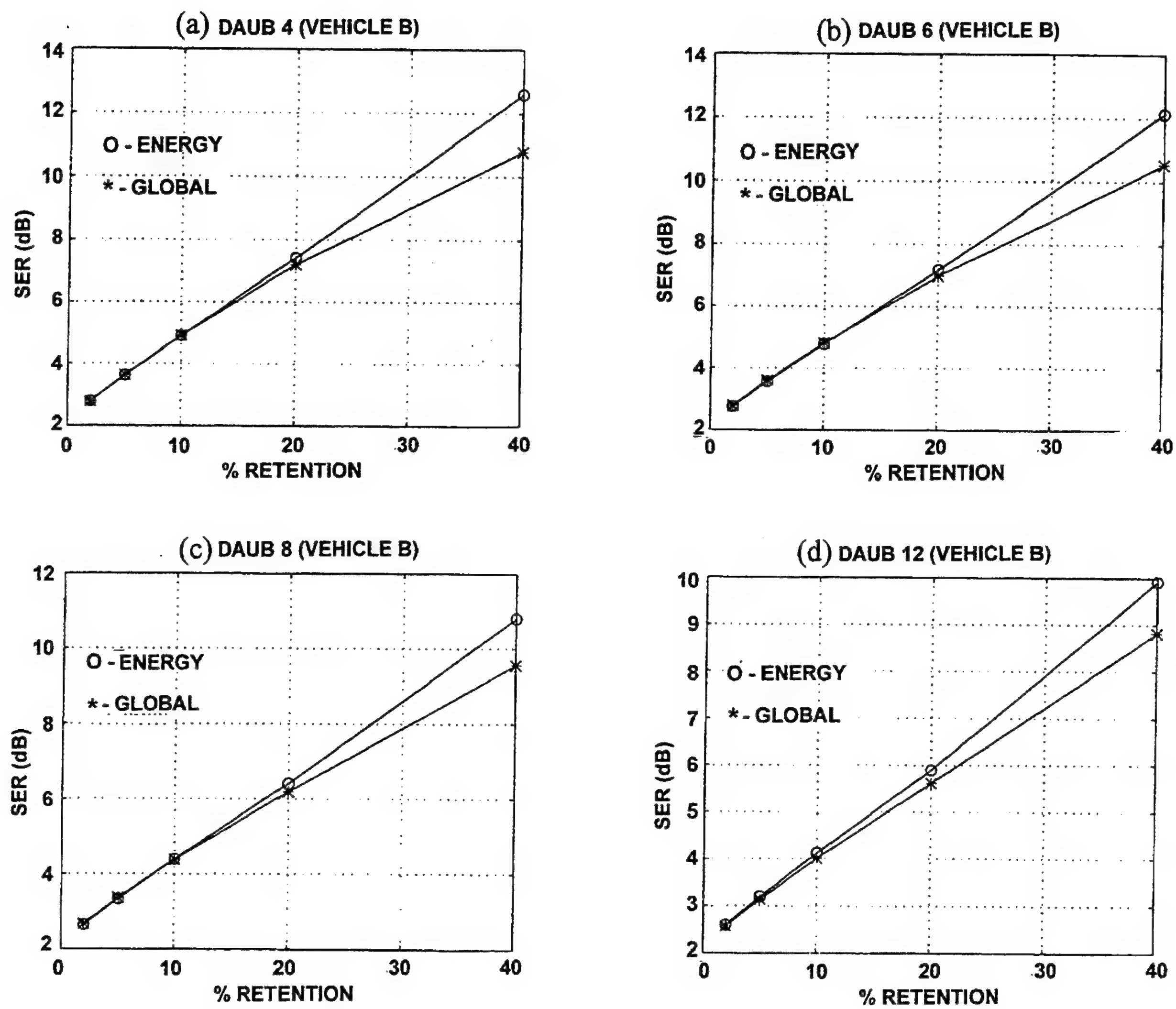

Figure 12. Reconstruction Performance Comparison for Vehicle B

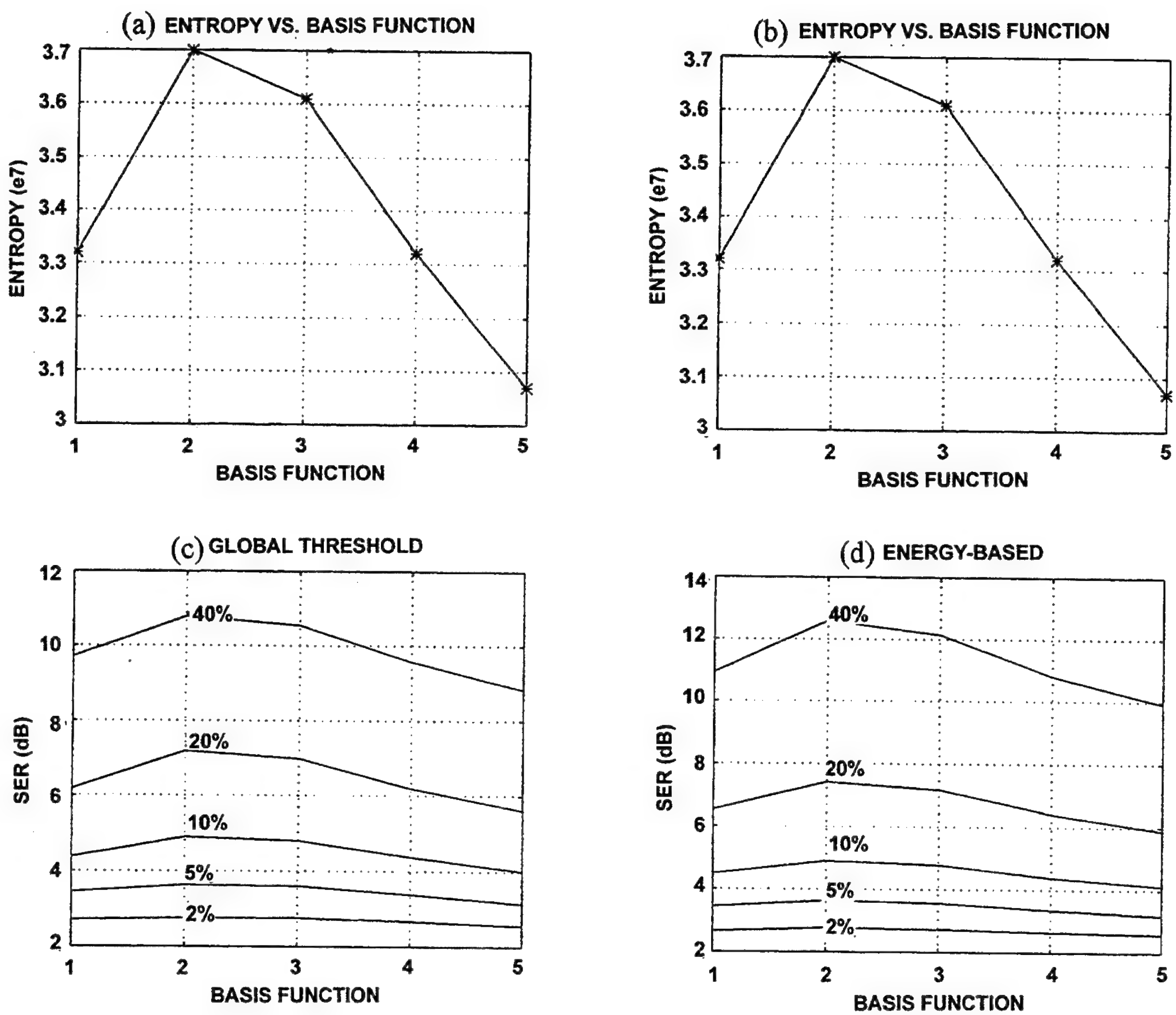

Figure 13. Reconstruction Performance of Vehicle-B Sound

Table 9. Distribution of Selected Wavelet Coefficients for Vehicle-A Signal

% Ret	2		5		10		20		40	
Level	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**
1	177	119	493	298	1040	596	2115	1193	4070	2386
2	151	125	324	313	588	626	1068	1253	1960	2505
3		41	2	103	10	206	86	412	440	823
4		22		54		108	8	215	79	430
5		11		28		57		113	5	226
6		5		12		23		46		93
7		2		6		12		24		48
8		1		3		6		11		23
9		1		2		4		7		15
10						1		1		3
11						1		1		2
12										
13										
14										
Total	*328*	*327*	*819*	*819*	*1638*	*1640*	*3277*	*3276*	*6554*	*6554*
SER	**0.54**	**0.60**	**1.13**	**1.32**	**1.98**	**2.40**	**3.58**	**4.44**	**6.82**	**8.46**

Table 10. Distribution of Selected Wavelet Coefficients for Vehicle-B Signal

% Ret	2		5		10		20		40	
Level	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**	**G**	**E**
1	322	224	801	560	1577	1119	3071	2239	5565	4478
2	6	61	12	151	61	302	206	605	935	1210
3		18		44		87		175	51	350
4		12		29		58		117	3	234
5		6		15		31		62		124
6		4		9		18		36		72
7		1		3		7		14		27
8		1		2		3		7		14
9		1		1		3		6		11
10		1		2		4		8		16
11		1		2		5		9		19
12										
13										
14										
Total	*328*	*330*	*819*	*818*	*1638*	*1637*	*3277*	*3278*	*6554*	*6555*
SER	**2.57**	**2.60**	**3.14**	**3.20**	**4.01**	**4.12**	**5.61**	**5.89**	**8.82**	**9.92**

Tables 9 and 10 show the wavelet coefficient selection distribution across decomposition levels for varying retention percentages (**% Ret**) using global thresholding (**G**) and mean-energy (**E**) selection methods. Signals were analyzed using the Daubechies 12-wavelet basis. The performance index of reconstruction is the SER in decibels.

5. CONCLUSIONS

A new mean energy-based wavelet coefficient selection technique for signal compression has been developed, and its performance has been compared with conventional global thresholding. The energy technique is a *local* thresholding method in that each level of the wavelet decomposition tree has its own independent threshold for coefficient selection; energy considerations provide the mechanism for determining this threshold. In contrast, global thresholding utilizes a universal threshold across all resolution levels to select coefficients for representation of the compressed signal. This conceptual difference is schematically illustrated in figure 14.

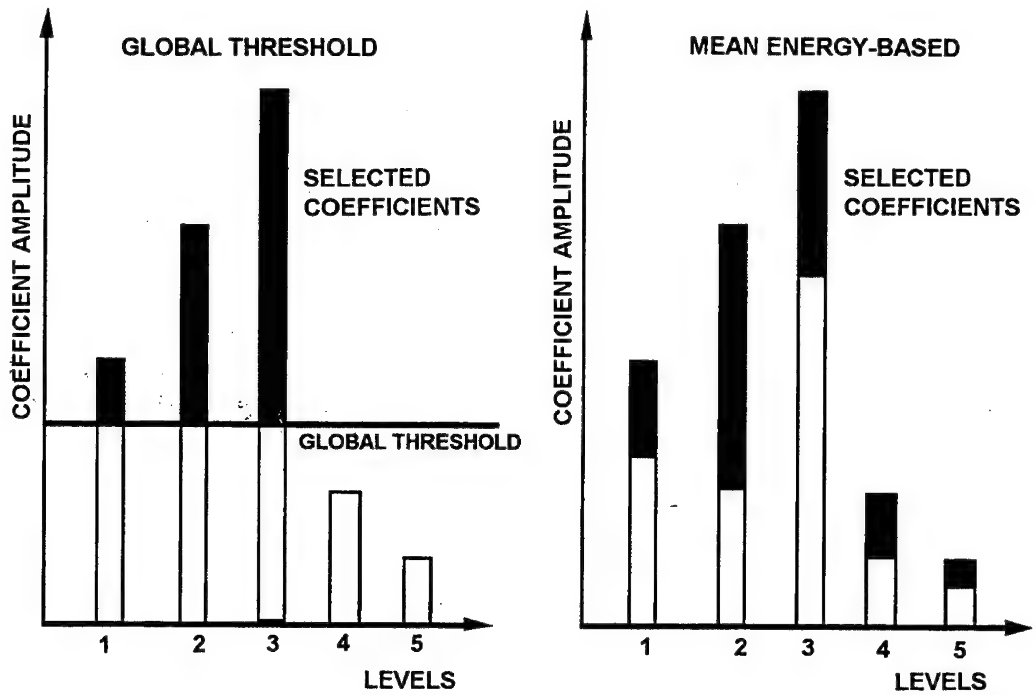

Figure 14. Conceptual Schematic for Global Threshold vs. Mean Energy-Based Wavelet Coefficient Selection Methods

The primary advantage of the energy method is that it tends to select wavelet coefficients from a broader spectrum of levels than the global thresholding method does. In particular, the energy method is biased towards selecting coefficients from deeper in the decomposition tree since the mean energy tends to increase with increasing level number. This occurrence is because total number of coefficients at level $k = (N / 2^k)$ decreases as k increases, tending to push up the overall average. This phenomenon is observed in the coefficient selection distributions tabulated in tables 1 and 2 (Daubechies four analysis) and tables 5 and 6 (Daubechies 12 analysis) for porpoise and dolphin signals.

A comparison of signal reconstruction performance between the two methods (figures 6 and 9) shows that the energy method always outperforms global thresholding at low-retention percentages. However, as the retention percentage increases, there may be a *crossover* ; from which point global thresholding performs better than mean energy-based selection (as seen in figure 9 (b) through (d) for dolphin signal reconstruction, where crossover occurs between 10-percent and 20-percent retention for Daubechies 6, 8, and 12 basis functions.

The wavelet coefficient-selection distribution across resolution levels is plotted in figures 15 and 16 for a Daubechies 12 decomposition of the porpoise and dolphin signals (reference tables 5 and 6). The distributions for the porpoise are shown in figure 15 and exhibit a marked difference between the two methods in both overall shape and depth in the decomposition tree. The energy-based distribution has a "heavy tail," which is the expected characteristic signature of the method and is the advantage over global thresholding. In contrast, figure 16 shows that the distributions for both selection methods for the dolphin tend to have the same overall shape and depth in the decomposition tree with increasing retention percentage. In such a situation, the primary advantage of the energy-based selection over global thresholding is nullified. If the two methods reach the same depth of the decomposition tree, global thresholding performs better because it is selecting the largest coefficients across the limited range of levels under consideration.

Figure 17 shows the reconstructed signal from energy-based compression compared to the reconstruction from global thresholding for the dolphin sound (using 20-percent retention of the Daubechies 12 basis). In the first phase of the signal, the energy-based reconstruction tracks the detailed variations of the original signal, whereas global thresholding has only 1 low-frequency component (see figure 17 (a)). However, once the signal enters its high-amplitude region, global thresholding follows the large peaks more closely than the energy method, which overall results in a better SER performance index for global thresholding versus the energy method (12.55 dB versus 11.09 dB).

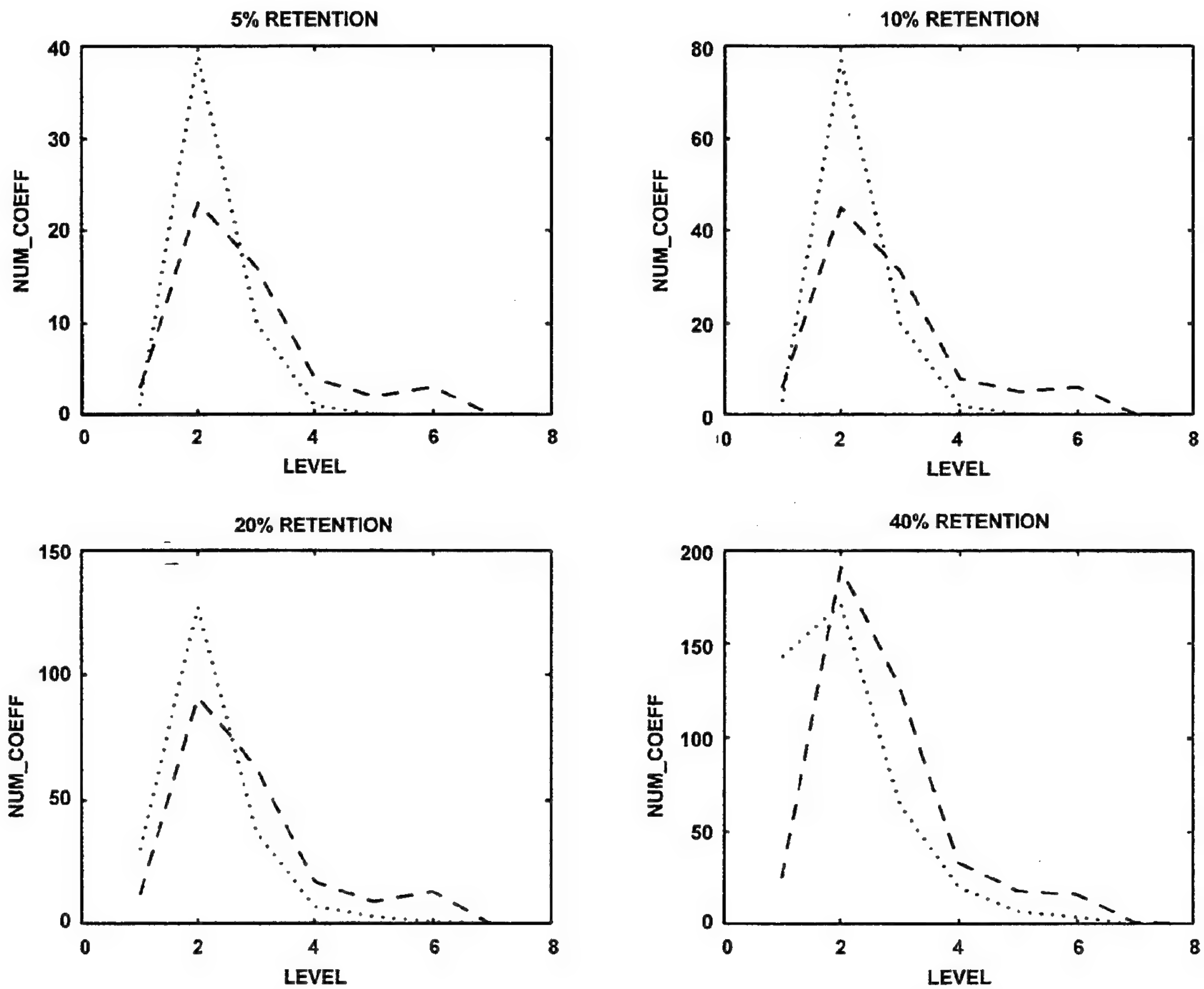

Figure 15. Wavelet Coefficient Distribution: Porpoise Sound-Daubechies 12 Basis
(Global ..... Energy -----)

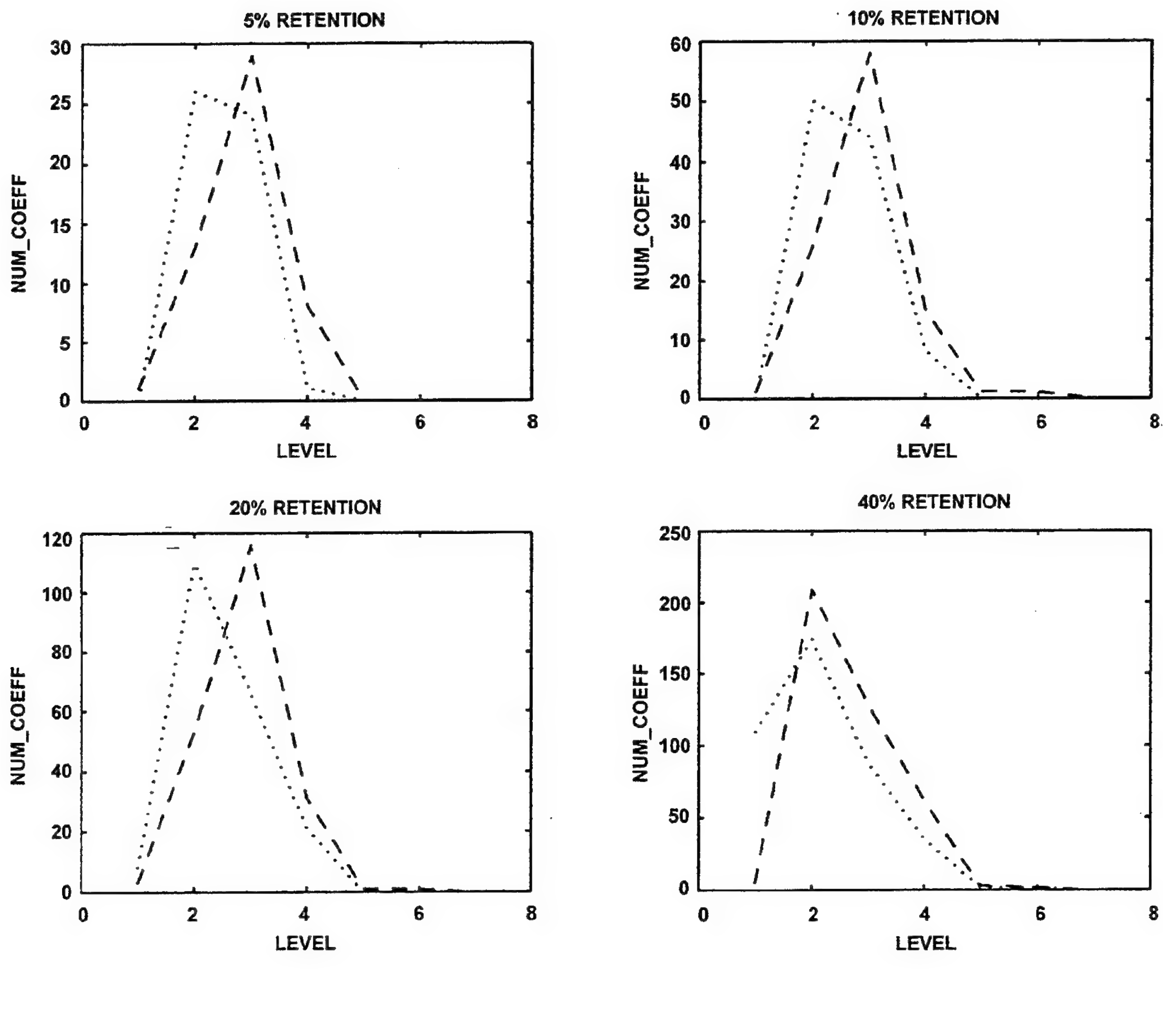

Figure 16. Wavelet Coefficient Distribution: Dolphin Sound-Daubechies 12 Basis (Global ..... Energy -----)

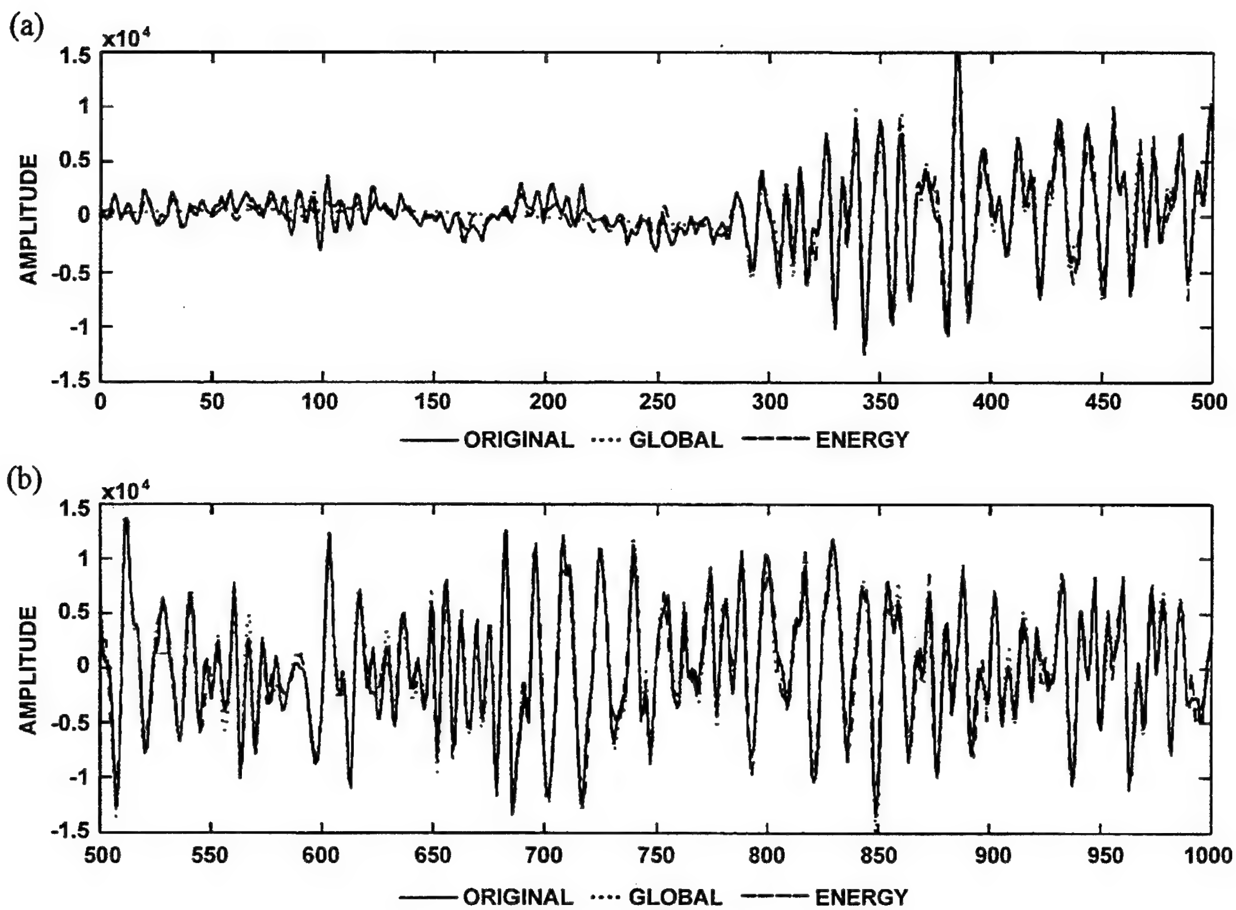

Figure 17. Signal Reconstruction: Dolphin Sound with 20-Percent Retention Using Daubechies 12 Basis

6. FUTURE WORK

The crossover effect is particularly pronounced in signals that are nonstationary in nature, such as speech. One possible approach to data compression in such a situation is *segmentation* wherein the signal is decomposed into segments of similar characteristics. Each segment would then be analyzed with a unique wavelet basis function that is optimal with respect to that segment. Open issues are (1) the choice of segmentation boundaries (such as points where the signal is determined to change characteristics) and (2) a real-time, adaptive segmentation strategy.

A preliminary analysis was conducted on the dolphin signal assuming a segmentation boundary between the initial low-amplitude region and the primary high-amplitude segment as shown in figure 18. These segments were analyzed independently, and the reconstruction performance on the total signal was determined for similar basis functions on the two segments. These performance curves are shown in figure 19, and it can be seen that the crossover effect has been greatly reduced from the original analysis in figure 10. Similar types of segmentation approaches have been extensively used in speech-processing (Wickerhauser, 1992). The foregoing analysis shows demonstrated potential for undersea applications and calls for future investigation.

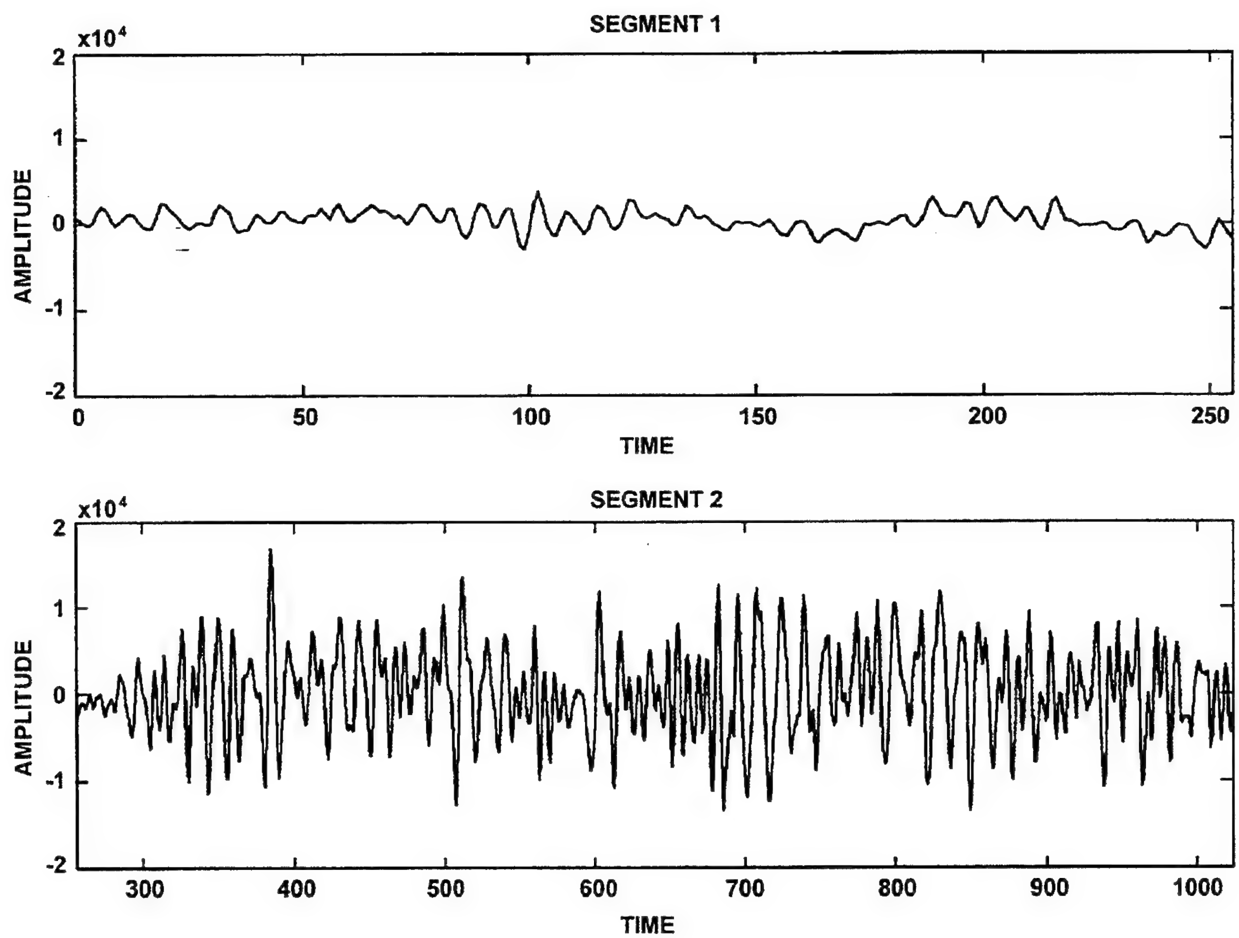

Figure 18. Signal Segmentation: Dolphin Sound

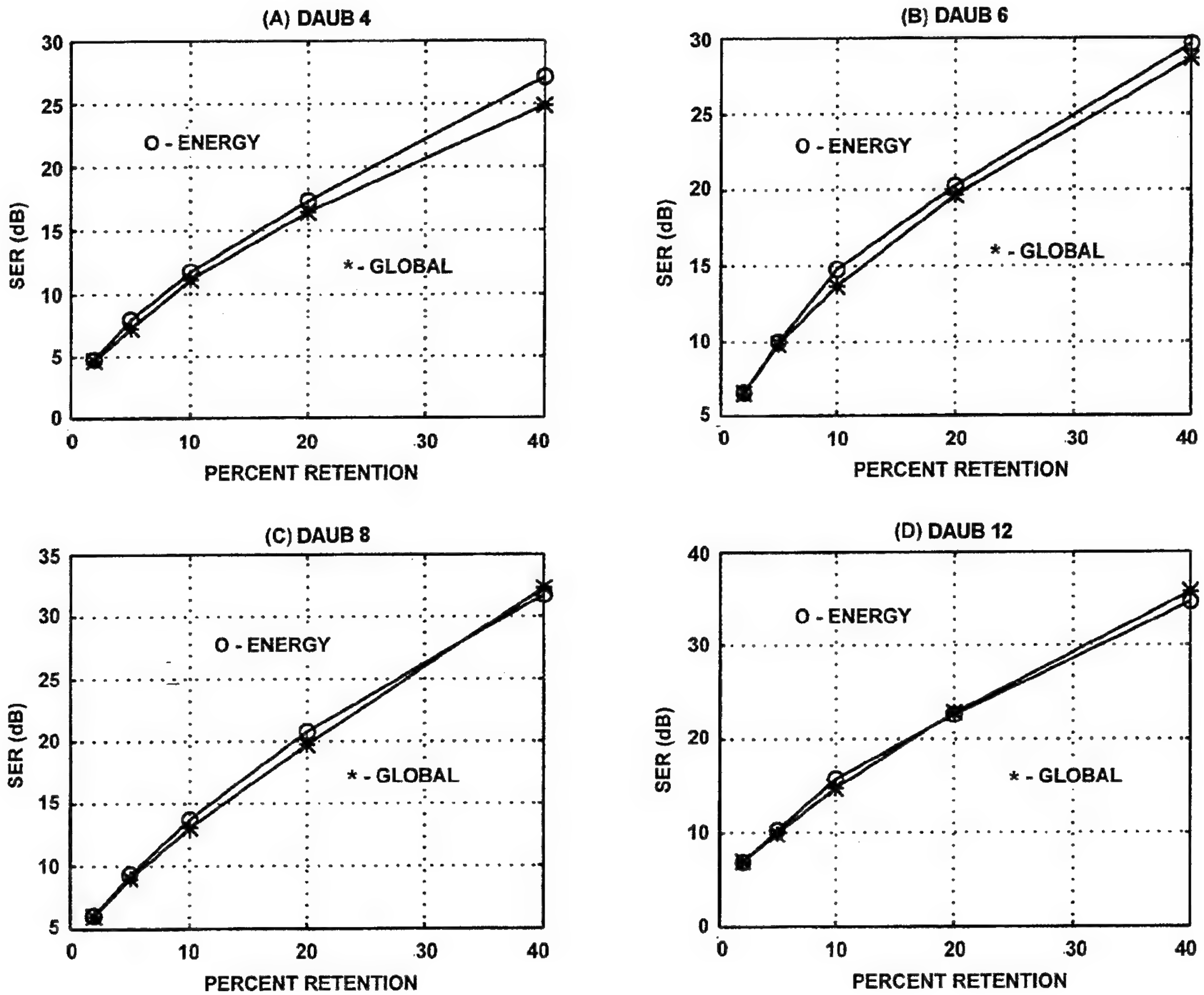

Figure 19. Reconstruction Performance: Dolphin Sound Analyzed in Two Segments

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APPENDIX

WAVELET TOOLKIT SOFTWARE

To understand and perform wavelet analysis, a software package called the Wavelet Toolkit has been developed at the Naval Undersea Warfare Center (NUWC). The Wavelet Toolkit is used to perform experiments on one-dimensional acoustic signals for data compression and reconstruction. A block diagram of the Wavelet Toolkit is given in figure A-1 and provides an overview of the system functionality. This program consists of three modules: wavelet decomposition, wavelet coefficient-selection techniques for data compression, and signal reconstruction.

Module 1-This part of the software decomposes a given time series into its wavelet coefficients. The user-specified input data file is assumed to have two columns: time and data. Given a signal with number of data points $N=2^k$, the decomposition is performed using the fast wavelet transform (FWT) algorithm as described in a journal article by Cody (Cody, 1992).

Module 2-Four different methods are implemented for selection of wavelet coefficients for signal data compression: level-based pruning, global threshold, local threshold, and mean energy-based.

(1) In the *level-based pruning* method, user-specified number of levels of the wavelet tree are removed, beginning from level 1 and proceeding downward. This method is a very elementary compression technique, somewhat arbitrary, and does not perform well.

(2) In the *global threshold* method, user-specified percentage of coefficients are retained, and largest coefficients in an overall global sense are selected. This method is a conventional method and is commonly used.

(3) In the *local threshold* method, user-specified percentage of coefficients are retained at each level, and largest coefficients in a local sense are selected. This method does not perform as well as conventional global thresholding.

(4) In the *mean energy-based* method, user-specified percentage of coefficients are retained. Criterion of selection is the mean energy at each level of the decomposition tree. The number of coefficients selected is proportional to the mean energy. In case of coefficient overflow, remaining coefficients are selected from levels that have available coefficients.

Module 3-Reconstructs the signal from the given wavelet coefficients. The user-specified output file has two columns: time and data.The following enhancements have been incorporated, debugged, and tested:

1. dynamic memory allocation for wavelet decomposition tree,
2. quicksort coefficient sorting algorithm,

3. performance index as defined by SER,
4. entropy of wavelet transform,
5. inclusion of Daubechies 2 through Daubechies 20 coefficients to 12-decimal place accuracy for comparative performance analysis,
6. performance calibration assuming a different number of coefficients retained,
7. best wavelet basis selection using minimum entropy.

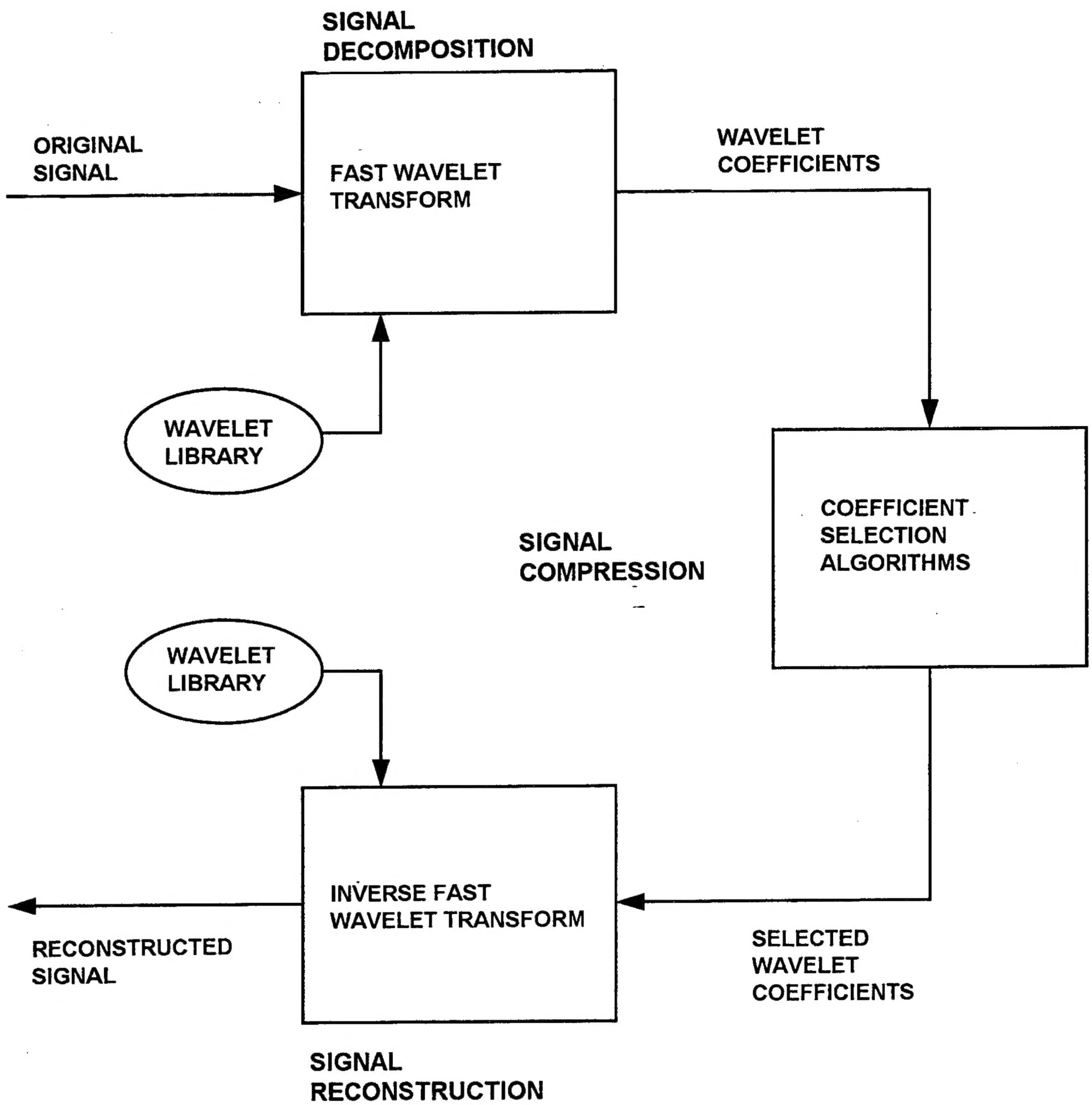

Figure A-1. Wavelet Toolkit Software System Block Diagram

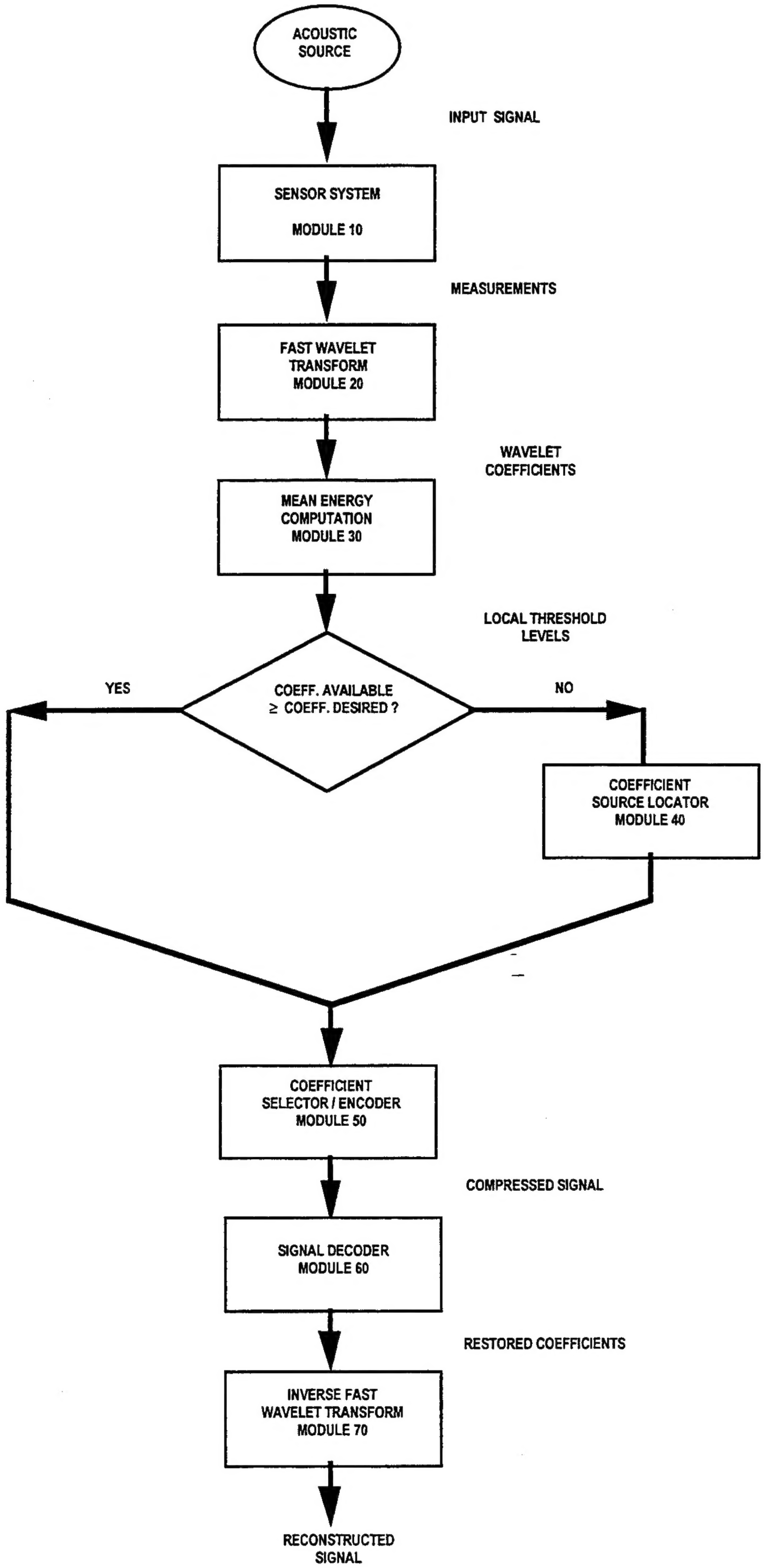

Figure A-2. Operational Flowchart of Signal Compression/Reconstruction System with Wavelets

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